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NUMERICAL SOLUTION OF ELECTROMAGNETIC SCATTERING PROBLEMS

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ABSTRACT

The purpose of this work is to describe a theoretical formulation, including a documented computer program, for the evaluation of electromagnetic scattering by perfectly conducting bodies having an axis of rotational symmetry. The main body of the work gives the theory, which has been modified considerably from that given earlier. Appendix I gives the analysis and logic which forms the basis for the various subroutines of the computer program. Appendix II gives the complete FORTRAN listings of the computer program. Finally, Appendix III gives the computer print-out for a numerical example, scattering by a conducting sphere-cone-sphere obstacle , as obtained on the IBM 7030 digital computer.

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SECTION I

INTRODUCTION

GENERAL DISCUSSION

In recent years work has begun to appear in the literature on the numerical solution of electromagnetic scattering problems by digital computer. For the most part these methods have involved numerical solution of a vector surface integral equation. In any case, the basic procedure in all methods requires numerical generation of the elements of an N $\times$ N matrix, followed by subsequent inversion. Because N increases roughly linearly with the size of the target (quadratically for bodies that are not axially symmetric) there are practical limitations on the sizes that can be treated successfully. Hence such computations, exact in the sense that in principle any desired accuracy may be attained, are extremely useful in the Rayleigh region and some portion of the resonance region, but must ultimately be supplemented by high frequency approximate techniques in order to obtain the complete frequency response of a given target. Observe that exact numerical computations may play a useful role in establishing the usefulness and accuracy of approximation techniques, and also in providing experimental targets for more comprehensive range calibration than is presently possible using spheres and dipoles.

An exact formulation of scattering of electromagnetic waves by perfectly conducting obstacles was given in 1949 by Maue, who obtained a pure integral equation and, alternatively, an integro-differential

equation, either of which suffices for determination of the unknown surface currents on the obstacle.[1] Both equations have been discussed in the excellent review article on diffraction by Hönl, Maue, and Westpfahl (HMW),[2] and a derivation of the pure integral equation has been presented by Van Bladel.[3] The integro-differential equation has been programmed and solved numerically on the digital computer by Andreasen,[4] who considered axially symmetric targets. Similarly, numerical analysis has been performed by Oshiro and co-workers, employing the pure integral equation for more general shapes.[5]

An alternative theoretical approach, also leading to numerical results, has been given by Waterman.[6] The purpose of this paper is to document a computer program for the implementation of this method. Section II gives the theory, which has been modified considerably from that given earlier. Appendix I gives the analysis and logic which forms the basis for the various subroutines of the computer program. Appendix II gives the complete FORTRAN listing of the computer program. Finally, Appendix III gives the computer printout for a numerical example, scattering by a conducting sphere-cone-sphere obstacle.

COMPUTATIONAL ASPECTS

In addition to their role in the present work, it should be noted that certain of the subroutines contained in this report may be of interest for other applications.

Principal among these are those routines for generating the spherical Bessel and Hankel functions by a combination of power series and recursion techniques, noting that both precision checks and alternative procedures are included for those cases where precision is difficult to maintain. The subroutine for generating associated Legendre functions, and their derivatives, by recursion is also essentially self-contained. Finally, certain of the matrix processing operations, e.g., orthogonalization, may prove of use elsewhere, perhaps with modifications.

SECTION II

THEORY

MATRIX FORMULATION

Consider an incident electromagnetic wave $\underline{E}^i(\underline{r})$, $\underline{H}^i(\underline{r})$ impinging on the closed, perfectly conducting surface σ of Figure 1 in otherwise free space. It is assumed throughout that σ is sufficiently regular that Green's theorem is applicable, and that σ possesses a continuous single-valued normal $\hat{n}$ at each point. Only simple harmonic time dependence at angular frequency ω is considered; a factor exp $(-i\omega t)$ is suppressed in all field quantities. Field behavior is described by Maxwell's equations in the form

$$\underline{\nabla} \times \underline{\nabla} \times \underline{E} - k^2\underline{E} = 0, \tag{1}$$

with an identical equation governing $\underline{H}$. In these equations $k=\omega/c=2\pi/\lambda$ is the free-space propagation constant.

Because the surface conductivity is infinite on σ, no tangential components of electric field can be supported. Currents are induced in the surface, the electric field of which must precisely cancel the tangential components of $\underline{E}^i$ at each point on σ. HMW have given a representation of the fields for this problem in terms of surface current. After minor modification their formulas may be written[2]

$$\underline{E}(\underline{r}) = \underline{E}^i(\underline{r}) + \int d\sigma'\underline{\nabla} \times \underline{\nabla} \times \underline{j}(\underline{r}')g_0\,(k|\underline{r}-\underline{r}'|), \tag{2a}$$

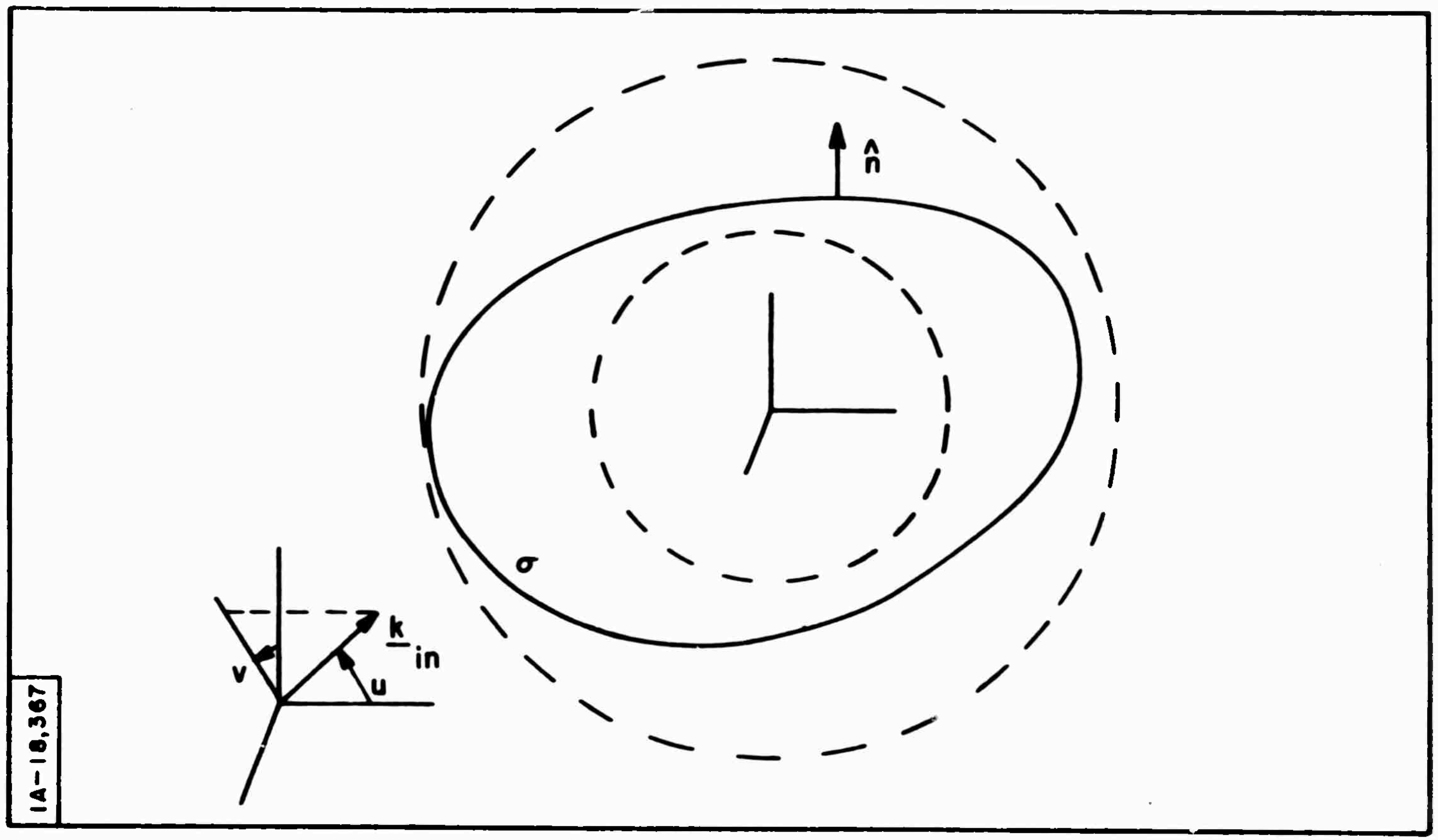

Figure 1. Geometry for a Plane Wave, Propagation Vector $\underline{k}_{in}$, Illuminating a Perfectly Conducting Obstacle Bounded by the Surface σ

$$\underline{H}(\underline{r}) = \underline{H}^{i}(\underline{r}) - ik \int d\sigma' \underline{\nabla} \times \underline{j}(\underline{r}') g_{o} (k|\underline{r}-\underline{r}'|), \tag{2b}$$

where $\underline{E}$, $\underline{H}$ is the total field, $g_o(kR) = (4\pi R)^{-1} \exp(ikR)$ is the (scalar) free space Green's function appropriate to outgoing waves, and the curl operators are with respect to the unprimed (i.e., not the integration) variables. The integrals represent the $\underline{E}$ and $\underline{H}$ fields, respectively, due to a surface distribution of electric dipoles, as one would anticipate on physical grounds. The quantity $\underline{j}(\underline{r})$, which we identify with induced surface current, stands for the jump discontinuity in magnetic field encountered in crossing the surface, i.e.

$$\underline{j} = -(1/ik)\hat{n} \times [\underline{H}_{+} - \underline{H}_{-}] \text{ on } \sigma \quad . \tag{2c}$$

In the course of obtaining Eqs. (2), the boundary conditions appropriate to conducting surfaces were employed, namely, that $n \times \underline{E}_{+} = n \times \underline{E}_{-} = 0$ on σ.

The nature of jump discontinuities in the field vectors across σ can be shown directly from Eqs. (2), giving

$$\underline{E}_{+} - \underline{E}_{-} = (\nabla_s \cdot \underline{j})\, \hat{n} = \hat{n} i \omega \rho \tag{3a}$$

$$\underline{H}_{+} - \underline{H} = ik\hat{n} \times \underline{j} \quad . \tag{3b}$$

In the first of these equations, the surface divergence[7] $\nabla_s \cdot \underline{j}$ of the current may be defined by the physical requirement that it equal the net flow of charge out of infinitesimal element of area (per unit area per unit time). The second equality, involving the surface charge density ρ, follows from the continuity equation $\nabla_s \cdot \underline{j} = - \partial\rho/\partial t$.

The extended boundary condition, requiring that the total electromagnetic field vanish identically in the interior (thus in particular $\underline{E}_- = 0$ on σ), is from Eq. (3a) sufficient to guarantee the usual exterior boundary condition $\hat{n} \times \underline{E}_+ = 0$. Applying the extended boundary condition in Eq. (2a) gives

$$\int d\sigma' \nabla \times \nabla \times \underline{j}(\underline{r}') g_0 (k|\underline{r}-\underline{r}'|) = - \underline{E}^i(\underline{r}) \quad , \tag{4}$$

an "extended" integral equation that is to hold for all points $\underline{r}$ in the small dashed sphere in Figure 1. By taking the curl of both sides of this equation, it follows that the total magnetic field $\underline{H}$ will also vanish in this region, once Eq. (4) is satisfied. Equation (4) is equivalent to three scalar equations for the two unknown tangential components of $\underline{j}$; only two of the equations are independent, however, in consequence of the fact that each side of Eq. (4) must have zero divergence.

Equation (4) may be satisfied by expanding both sides in regular vector eigenfunctions[8] $\underline{M}_{\sigma mn}$, $\underline{N}_{\sigma mn}$ of the vector Helmholtz Eq. (1).

To treat the integral one writes $\underline{j} g_0 = \underline{j} \cdot \underline{\underline{1}} g_0$; the expansion of the "free space Green's dyad" $\underline{\underline{1}} g_0$ has been given by Morse and Feshbach.[9] Because of orthogonality over any spherical surface about the origin shown in Figure 1, corresponding coefficients may be equated on both sides of Eq. (4) to give, for incident plane $\underline{E}^i(r) = \hat{e}_0 e^{ik\cdot r}$,

$$\int d\sigma \underline{j}(\underline{r}) \cdot \underline{M}^3{}_{\sigma mn}(\underline{r})$$

$$= - (4\pi/ik^3) i^n [n(n+1)]^{\frac{1}{2}} \hat{e}_0 \cdot \underline{C}_{mn}{}^{\sigma}(\hat{k}) \quad ,$$

$$\int d\sigma \underline{j}(\underline{r}) \cdot \underline{N}^3{}_{\sigma mn}(\underline{r})$$

$$= + (4\pi/ik^3) i^n [n(n+1)]^{\frac{1}{2}} \hat{e}_0 \cdot i\underline{B}_{mn}{}^{\sigma}(\hat{k}) \quad . \tag{5}$$

The $\underline{M}^3$, N^3 are the outgoing wave functions, and dependence on the direction of incidence $\hat{k}$ is contained in the vector spherical harmonics $\underline{C}_{mn}{}^{\sigma}$, $\underline{B}_{mn}{}^{\sigma}$.[10] These equations are to hold for each triplet of values (σ, m, n), with $\sigma = e, o$ (even, odd), $m = 0, 1, \ldots, n$, $n = 1, 2, \ldots$. These are the conditions under which the total $\underline{E}$, $\underline{H}$ field will vanish identically in that volume consisting of the largest sphere inscribable within σ about the coordinate origin employed. As has been shown elsewhere,[6] because of analytic continuability this is adequate to guarantee that $\underline{E}$ and $\underline{H}$ will vanish identically throughout the entire interior volume.

The surface current is next approximated by expansion in the assumed complete set of tangential vector functions $\hat{n} \times \underline{M}$ and $\hat{n} \times \underline{N}$; one writes

$$\underline{j}(\underline{r}) = (4/ik)\sum_{\sigma'm'n'} [a_{\sigma'm'n'}\hat{n}(\underline{r}) \times \underline{M}_{\sigma'm'n'}(\underline{r})$$

$$+ b_{\sigma'm'n'}\hat{n}(\underline{r}) \times \underline{N}_{\sigma'm'n'}(r)] \qquad (6)$$

where the expansion coefficients remain to be determined. At this point one can expedite the discussion by introducing a matrix notation. First, the triplet of indices appearing in Eqs. (5) and (6) are regrouped into a single index ν by the ordering (σmn) = e01, o01, e11, o11, e02, ... The vector spherical harmonics may then be written as column matrices $\underline{C}$, $\underline{B}$, having as their νth elements $(i)^n[n(n + 1)]^{\frac{1}{2}}\, \underline{C}_{mn}^{\sigma}(\hat{k})$, and $(i)^n[n(n + 1)]^{\frac{1}{2}}\, \underline{B}_{mn}^{\sigma}(\hat{k})$, respectively. The undetermined expansion coefficients of Eq. (6) are simply designated by the column matrices a, b.

In this notation, substitution of the expansion Eq. (6) into Eq. (5) yields a pair of coupled matrix equations

$$\begin{bmatrix} I & J \\ K & L \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \hat{e}_o \cdot \underline{C} \\ -e_o \cdot i\underline{B} \end{bmatrix} \qquad (7)$$

for the determination of a and b. The matrix elements of I are given, after rewriting the triple scalar product that appears, by

$$I_{\nu\nu'} = (k^2/\pi) \int d\sigma \hat{n}(\underline{r}) \cdot \underline{M}^3_{\sigma mn}(\underline{r}) \times \underline{M}_{\sigma' m' n'}(\underline{r}) \quad , \tag{8a}$$

and the four matrices I, J, K, L differ from each other only in the vector products appearing in the integrand of Eq. (8) which are, respectively, $\underline{M}^3_\nu \times \underline{M}_{\nu'}$, $\underline{M}^3_\nu \times \underline{N}_{\nu'}$, $\underline{N}^3_\nu \times \underline{M}_{\nu'}$, and $\underline{N}^3_\nu \times \underline{N}_{\nu'}$. By inspection of the integrands, in view of the fact that $\underline{M} = \mathrm{Re}\, \underline{M}^3$ and $\underline{N} = \mathrm{Re}\, \underline{N}^3$, it is clear that ReI and ReL are skewsymmetric, whereas ReJ and ReK are symmetric. The surface integrals of Eq. (8a) must, in general, be done numerically and are most conveniently performed in spherical coordinates θ, ϕ, for which the appropriate radial coordinates to employ may be given by the parametric specification $r = r(\theta, \phi)$ of the surface. In view of Green's second vector identity

$$\int d\sigma \hat{n} \cdot [\underline{A} \times \nabla \times \underline{B} - \underline{B} \times \nabla \times \underline{A}] = \int d\tau \, [\underline{B} \cdot \nabla \times \nabla \times \underline{A} - \underline{A} \cdot \nabla \times \nabla \times \underline{B}]$$

the matrices may be seen to be interrelated by

$$K = - J + i(D_+)^{-1} \quad ,$$

$$L = - I \quad , \tag{8b}$$

where the <u>diagonal</u> matrix D_+ (and D_-, employed below) has νth elements defined by

$$(D_{\pm})_\nu \equiv (\pm 1)^n \frac{\mathcal{E}_m (2n+1)\ (n-m)!}{4n(n+1)\ (n+m)!} \quad . \tag{8c}$$

The Neumann factor $\mathcal{E}_m$ is given by $\mathcal{E}_o = 1$, $\mathcal{E}_m = 2$ otherwise.

It is also desired to compute the scattered field $\underline{E}^s$, $\underline{H}^s$ given by the surface integrals in Eq. (2). Specifically for the electric field, one has

$$\underline{E}^s(\underline{r}) = 4 \sum_{\sigma mn} [f_{\sigma mn} \underline{M}^3{}_{\sigma mn}(\underline{r})$$

$$+ g_{\sigma mn} \underline{N}^3{}_{\sigma mn}(\underline{r})]; \quad r > r'_{\text{max on } \sigma}$$

$$\sim \underline{F}(\hat{k}_{out}, \hat{k}_{in}) e^{ikr}/r; \quad kr \gg 1. \tag{9}$$

The vector scattering amplitude $\underline{F}$, depending both on direction of incidence $\hat{k}_{in}$ and observation $\hat{k}_{out}$, is obtained by introducing asymptotic forms of the outgoing partial waves $\underline{M}^3$, $\underline{N}^3$ in the preceding expression for $\underline{E}^s$ to get

$$\underline{F}(\hat{k}_{out}, \hat{k}_{in}) = (4/ik)\ [\underline{C}'(\hat{k}_{out})\ D_- f + iB'\ (\hat{k}_{out}) D_- g] \ , \tag{10}$$

where $\underline{C}'$ is the transpose of $\underline{C}$ (and hence a row matrix). The outgoing partial wave expansion coefficients f, g are expressed in terms of surface currents a, b by

$$\begin{bmatrix} f \\ g \end{bmatrix} = - \mathrm{Re} \begin{bmatrix} I & J \\ K & L \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad . \tag{11}$$

These formulas have been obtained by employing that expansion of the free space Green's dyad valid in the exterior region outside the large dashed sphere of Figure 1.

The scattering cross section σ^{scat} is given by[11]

$$\sigma^{scat} = (16\pi/k^2)\ (f'^{*} D\, f + g'^{*} D_{+} g) \quad . \tag{12a}$$

As a numerical check on accuracy, one may also compute the total cross section

$$\sigma^{tot} = (4\pi/k)\ \mathrm{Im}\ [\hat{e}_o \cdot \underline{F}\ (\hat{k}_{in},\ \hat{k}_{in})] \quad , \tag{12b}$$

which must equal σ^{scat} by the forward amplitude theorem[11]. The radar cross section, defined as 4π times the back-scattered power per steradian divided by incident power per unit area, is given by

$$\sigma^{radar} = (64\pi/k^2)\ |\hat{e}_o \cdot \underline{C}'\ (-\hat{k}_{in}) D_{-} f + i\hat{e}_o \cdot \underline{B}'\ (-\hat{k}_{in}) D_{-} g|^2 \quad . \tag{12c}$$

If the return signal is regarded as resolved into two orthogonal linearly polarized modes, then this equation gives a measure of the power carried in that mode having polarization aligned with the original incident wave, whereas the <u>cross-polarized</u> return is given by replacing $\hat{e}_o$ by $\hat{e}_o' = \hat{k}_{in} \times \hat{e}_o$ in Eq. (12c).

EVALUATION OF THE TRANSITION MATRIX

Instead of first solving Eq. (7) for the currents a, b, then substituting in Eq. (11) to obtain the scattered wave f, g, the currents may be formally eliminated to obtain the scattered wave directly from the incident wave as

$$\begin{pmatrix} f \\ g \end{pmatrix} = -\begin{pmatrix} D_+^{-\frac{1}{2}} & 0 \\ 0 & D_+^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix} \begin{pmatrix} D_+^{\frac{1}{2}} & 0 \\ 0 & D_+^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} e_o \cdot \underline{C} \\ -e_o \cdot i\underline{B} \end{pmatrix} . \tag{13}$$

The block matrix,

$$T \equiv \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix} , \tag{14}$$

is known as the <u>transition</u> matrix, and is both symmetric (i.e. $T_1' = T_1$, $T_2' = T_3$, $T_4' = T_4$) and has the property $T^{*\prime}T = \mathrm{Re}T$, i.e.,

$$\begin{pmatrix} T_1^* & T_2^* \\ T_3^* & T_4^* \end{pmatrix} \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix} = \begin{pmatrix} T_1^*T_1 + T_2^*T_3 & T_1^*T_2 + T_2^*T_4 \\ T_3^*T_1 + T_4^*T_3 & T_3^*T_2 + T_4^*T_4 \end{pmatrix} = \mathrm{Re} \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix} . \tag{15}$$

The property Eq. (15) is a consequence of unitarity of the scattering matrix $S = 1-2T$, as may be verified by substitution in the unitarity condition $S'^*S = 1$.

If one now defines the matrix Q as

$$Q = \begin{pmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{pmatrix} \equiv \begin{pmatrix} D_+^{\frac{1}{2}} & 0 \\ 0 & D_+^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} J' & L' \\ I' & K' \end{pmatrix} \begin{pmatrix} D_+^{\frac{1}{2}} & 0 \\ 0 & D_+^{\frac{1}{2}} \end{pmatrix} , \qquad (16)$$

then by comparison with Eqs. (7) and (11) the transition matrix is determined by the matrix equation

$$QT = \mathrm{Re}\, Q \quad , \qquad (17)$$

which in general must be solved numerically.

Instead of working with the 2 by 2 block form of Eq. (17), involving in truncation four $N \times N$ matrices, it is convenient, for the numerical processing, to change over to single $2N \times 2N$ matrices. Thus, define the $2N \times 2N$ matrix $\hat{Q}$ by

$$\left.\begin{aligned} \hat{Q}_{(2m-1)\,(2n-1)} &= (Q_1)_{mn} \\ \hat{Q}_{(2m-1)\,(2n)} &= (Q_2)_{mn} \\ \hat{Q}_{(2m)\,(2n-1)} &= (Q_3)_{mn} \\ \hat{Q}_{(2m)\,(2n)} &= (Q_4)_{mn} \end{aligned}\right\} \quad m, n = 1, 2, \ldots, N \ . \qquad (18)$$

The matrices $\hat{T}$, and $\hat{S} = 1 - 2\hat{T}$ are defined in exact analogy to this. At this point, Eq. (16b) may be written in terms of $\hat{S}$ as

$$\hat{Q}\hat{S} = -\hat{Q}^* \quad . \tag{19}$$

Because of the behavior of the radial (Hankel) functions that appear in the matrix elements of $\hat{Q}$, the imaginary parts of the elements of Q will tend to grow very large numerically above the diagonal. In order to avoid loss of precision due to the finite precision arithmetic employed by the digital computer, it is convenient at this stage to reset all the mentioned elements to zero, by Gaussian elimination. This process is straightforward, the net effect being to premultiply $\hat{Q}$ by a real upper triangular matrix (all elements zero below the main diagonal). Suppose this conditioning to have been performed on Eq. (19), which we continue to employ without change of notation.

To Eq. (19) are adjoined the constraints of symmetry and unitarity mentioned above, which are unaffected by the $S \rightarrow \hat{S}$ transformation and thus given by

$$\hat{S} = \hat{S}' \tag{20}$$

and

$$\hat{S}'^*\hat{S} = 1 \quad . \tag{21}$$

Two extremes of view with regard to the system of Eqs. (19), (20) and (21) are as follows: first, one might truncate the matrix Eq. (19), solve numerically by digital computer, then compare the resulting solution with Eqs. (20) and (21), the latter thus being employed as consistency checks. On the other hand, one might attempt to treat all three equations from a unified point of view from the onset, obtaining a solution in some sense of Eq. (19) subject to the constraints of Eqs. (20) and (21). The first approach has been employed in earlier work on the computer for bodies of rotational symmetry, and works quite satisfactorily for a restricted range of body shapes and sizes. The second approach is employed in the present work in order to extend the range of bodies that can be handled, in view of the fact that the constraints essentially determine three quarters of the solution [i.e., of the $8N^2$ real parameters appearing in the $2N \times 2N$ (truncated) complex matrix S, it can be shown that only $N(2N + 1)$ are independent, if $\hat{S}$ satisfies Eqs. (20) and (21)].

To develop a unified analysis, observe first that if $\hat{S}$ could be constructed in the form

$$\hat{S} = U' \, U \tag{22}$$

where U is unitary, then both constraints would be satisfied by inspection. This suggests that, rather than inverting $\hat{Q}$ directly in Eq. (19), it be

made unitary. Thus consider the upper triangular matrix M (i.e., all elements are zero below the main diagonal) which by premultiplication makes $\hat{Q}$ into a unitary matrix $\hat{Q}_{unit}$, viz.

$$M\hat{Q} = \hat{Q}_{unit} \quad . \tag{23}$$

Premultiplying Eq. (19) by M, one can write

$$\hat{Q}_{unit}\hat{S} = - M\hat{Q}^* = - MM^{*-1}\,\hat{Q}^*_{unit} \quad .$$

Upon solving for $\hat{S}$, there now results

$$\hat{S} = - \hat{Q}'^*_{unit}\,(MM^{*-1})\,\hat{Q}^*_{unit} \quad . \tag{24}$$

Substituting this result in Eq. (20), the symmetry constraint, it follows without difficulty that the matrix product MM^{*-1} must be symmetric. But each of the matrices appearing in the product is upper triangular, and their product is again upper triangular. Consequently the product must be a diagonal matrix. Further, the diagonal elements can be written out explicitly, giving

$$MM^{*-1} = \begin{bmatrix} M_{11}/M_{11}^* & 0 & \cdots \\ 0 & M_{22}/M_{22}^* & \cdots \\ 0 & 0 & \\ \cdot & & \\ \cdot & & \\ \cdot & & \end{bmatrix} \quad .$$

If next one can arrange to choose the diagonal elements of M to be real, then

$$MM^{*-1} = 1 \quad . \tag{25}$$

From Eq. (24) the $\hat{S}$-matrix is now given by

$$\hat{S} = - \hat{Q}'^{*}_{unit} \hat{Q}^{*}_{unit} \quad , \tag{26}$$

which is of the required form Eq. (22). Substituting Eq. (26), along with the identity $\hat{Q}'^{*}_{unit}\hat{Q}_{unit} = 1$, back in the relation $\hat{S} = 1 - 2\hat{T}$, the desired transition matrix is given by

$$\hat{T} = \hat{Q}'^{*}_{unit} \, \mathrm{Re} \, (\hat{Q}_{unit}) \quad , \tag{27}$$

and the block form of T is readily obtained by reversing the transformation of Eq. (18).

Returning to M for a moment, Eq. (25) states simply that M is real. Thus the process may be summed up in the (formal) theorem: given the matrix Eq. (19), with constraints, Eq. (20) and (21), on the solution, it follows that the given matrix Q cannot be arbitrary, but must be such as to be factorizable into the product of a <u>real</u> upper triangular matrix and a unitary matrix, namely

$$\hat{Q} = M^{-1} \hat{Q}_{unit} \quad . \tag{28}$$

The transformation of $\hat{Q}$ into a unitary matrix, as required in Eq. (23), is done by Schmidt orthogonalization of the 2N vectors given by the rows of $\hat{Q}$, beginning with the bottom row and working up. The procedure is straightforward, and details are described in a subsequent section.

APPLICATION TO SPECIAL GEOMETRIES

In order to apply the equations to bodies having an axis of rotational symmetry, the axis of symmetry is chosen as polar axis for our spherical coordinates and, without loss of generality, the direction of incidence taken in the plane of azimuth $v = 0$, so that $\hat{k}_{in} = \hat{k}_{in}(u, 0)$. A reduced index notation may be employed for those matrix elements that do not vanish under the azimuthal integration, writing

$$
\begin{aligned}
I_{mnn'} &\equiv I_{omnemn'} = - I_{emnomn'} \\
J_{mnn'} &\equiv J_{emnemn'} = J_{omnomn'} \\
K_{mnn'} &= - J_{mnn'} + i\delta_{nn'} D_{mnn}^{-1} \\
L_{mnn'} &= I_{mnn'} \quad .
\end{aligned}
\tag{29}
$$

The independent matrix elements, written out, are

$$I_{mnn'} \equiv m \int_0^{\pi} d\theta \frac{\partial}{\partial\theta} (P_n^m P_{n'}^m) (kr)^2 h_n(kr) j_{n'} (kr)$$

$$J_{mnn'} \equiv \frac{-2}{\epsilon_m} \int_0^{\pi} d\theta \sin\theta \left[\frac{m^2 P_n^m P_{n'}^m}{\sin^2\theta} + \frac{\partial P_{n'}^m}{\partial\theta} \frac{\partial P_{n'}^m}{\partial\theta} \right] krh_n(kr) \frac{d}{dkr} [krj_{n'}(kr)] \tag{30}$$

$$\frac{-2}{\epsilon_m} n'(n' + 1) \int_0^{\pi} d\theta \sin\theta \frac{\partial P_n^m}{\partial\theta} P_{n'}^m h_n(kr) j_{n'}(kr) \frac{\partial}{\partial\theta} (kr)$$

Observe that the real parts of all these matrices are symmetric. Also, because of the vanishing of all matrix elements with different azimuthal mode indices ($m \neq m'$), there is no coupling and each azimuthal mode $m = 0, 1, 2, \ldots$ may be evaluated separately.

From the defining Eq. (16), the only non-vanishing elements of the Q matrix may now be written in reduced index notation as

$$\begin{aligned}
(Q_1)_{mnn'} &\equiv (Q_1)_{emnemn'} = (Q_1)_{omnomn'} \\
(Q_2)_{mnn'} &\equiv (Q_2)_{omnemn'} = -(Q_2)_{emnomn'} \\
(Q_3)_{mnn'} &\equiv (Q_3)_{emnomn'} = -(Q_3)_{omnemn'} \\
(Q_4)_{mnn'} &\equiv -(Q_4)_{emnemn'} = -(Q_4)_{omnomn'}
\end{aligned} \tag{31}$$

In addition, the reduced index elements are related by

$$(Q_3)_{mnn'} = (Q_2)_{mnn'}$$

$$(Q_4)_{mnn'} = -(Q_1)_{mnn'} + i \underline{\underline{1}} \quad . \tag{32}$$

Finally, examination of Eq. (17) reveals that the non-vanishing elements in the four blocks of the T matrix are interrelated exactly as in Eq. (31), but not Eq. (32), so that the complete solution may be obtained by solving Eq. (17) once, using the reduced index quantities.

A further important reduction occurs in the preceding equations for obstacles (e.g., finite cylinder) having a plane of mirror symmetry normal to the axis of rotational symmetry. For this geometry the radius vector $r(\theta)$ specifying the shape of the obstacle will be even about $\theta = \pi/2$, i.e.,

$$r(\theta) = r(\pi - \theta) \quad . \tag{33}$$

Inspection of the parity of the integrands giving rise to matrix elements in Eq. (30) readily reveals that a checkerboard pattern of zeros has emerged, i.e.,

$$I_{mnn'} = 0; \ (n + n') \text{ even}$$

$$J_{mnn'} = 0; \ (n + n') \text{ odd} \quad . \tag{34}$$

These elements can hence be set to zero without performing the numerical integrations.

Prolate (and oblate) spheroids have a mirror symmetry plane normal to their rotational symmetry axis, so that both mode and parity decompositions may be made, as discussed above. There is another reduction that occurs here, however, which from a theoretical standpoint lays the Rayleigh expansion out in full view, and for numerical purposes yields extremely well-conditioned matrices for inversion.

To see this, let us examine the matrix elements as given by Eqs. (30). The numerical magnitude of these elements is influenced mainly by the radial functions appearing in the integrand. For $I_{mnn'}$, for example, one has

$$I_{mnn'} \sim (kr)^2 \, h_n(kr) \, j_{n'}(kr) \equiv (kr)^2 \, [j_n j_{n'} + i \, n_n j_{n'}] \quad .$$

For a given argument x, the Bessel functions $j_n(x)$ decrease rapidly in magnitude, and the Neumann functions $n_n(x)$ increase, roughly as soon as the index n exceeds x. Thus the real part of I, which is obviously symmetric, will eventually decrease rapidly in magnitude as one proceeds along any row or column. The numerical behavior of I is dominated by its imaginary part, for which elements again decrease going out any row, but <u>increase</u> going down any column, at such a rate that diagonal elements remain relatively constant. These large numerical values presumably strongly influence the truncated matrix inversion procedure.

One can show, however, that for prolate or oblate spheroids this behavior, specifically the arbitrarily large values by which elements of I below the diagonal exceed corresponding elements above, vanishes identically. I and J become completely symmetric, and dominant terms lie only on the diagonal once either row or column index exceeds kr_{max}, where r_{max} is the radius of the circumscribing sphere.

Based on results given by Watson[12] one can show that the radial factor for an element below the diagonal in the imaginary part of $I_{mnn'}$ is of the form

$$x^2 n_{n+2s+1} j_n \doteq x^2 n_n j_{n+2s+1} + \frac{1}{x^{2s}} + \frac{1}{x^{2s-2}} + \cdots + 1 \qquad (35)$$

where the equivalence symbol ($\doteq$) indicates that the exact coefficients of inverse powers of x^2 have not been included, as they are not required in the present discussion. The first term on the right-hand side corresponds precisely to the symmetrically placed element above the diagonal; we must thus show that the inverse powers of x^2 contribute nothing to the integral

$$\mathrm{Im}\left[I_{m(n+2s+1)n}\right] = m \int_0^{\pi} d\theta \, \frac{\partial}{\partial\theta} \left(P^m_{n+2s+1} P^m_n\right) (kr)^2 n_{n+2s+1}(kr) \, j_n(kr) \quad . \qquad (36)$$

For a prolate (oblate) spheroid, having semi-axes a, b, one has

$$kr = ka\,[\cos^2\theta + (a/b)^2 \sin^2\theta]^{-\frac{1}{2}} \quad , \tag{37}$$

which may be rewritten (identifying x with kr)

$$1/x^2 \doteq P_o + P_2 \quad . \tag{38}$$

Now the product of two Legendre functions may itself be expanded in a series of Legendre polynomials, with indices ranging from the difference to the sum of the original indices,[13] i.e.,

$$P_n^m P_{n'}^m \doteq \sum_{\substack{p=n-n' \\ (p+n+n' \text{ even})}}^{n+n'} P_p \tag{39}$$

where again explicit numerical coefficients have been ignored. Substituting Eq. (38) in the series of inverse powers of x^2 appearing in Eq. (35), then employing Eq. (39) repeatedly, one can write

$$x^2 n_{n+2s+1} j_n \doteq x^2 n_n j_{n+2s+1} + \sum_{q=0}^{s} P_{2q} \quad . \tag{40}$$

This result may be put in Eq. (36), recalling also that ReI is symmetric, to get

$$I_{m(n+2s+1)n} - I_{mn(n+2s+1)} \doteq m \int_0^\pi d\theta \frac{\partial}{\partial\theta} (P^m_{n+2s+1} P^m_n) \sum_{q=0}^{s} P_{2q}$$

$$\doteq -m \int_0^\pi d\theta \sin\theta \, P^m_{n+2s+1} P^m_n \sum_{q=1}^{s} P_{2q-1}$$

$$\doteq -m \int_0^\pi d\theta \sin\theta \sum_{p=s}^{s+n} P_{2p+1} \sum_{q=1}^{s} P_{2q-1}$$

$$= 0 \qquad , \qquad (41)$$

where in the second step we have integrated by parts, then employed Eq. (39), and finally observed that the highest Legendre polynomial appearing in the second sum is P_{2s-1}, while the first sum begins at P_{2s+1}; because of orthogonality, all the resulting integrals vanish.

To perform the analogous calculation for $J_{mnn'}$, one proceeds by first employing Green's identity to rewrite $J_{mnn'}$ in the more symmetric form

$$J_{mnn'} = -\frac{1}{\epsilon_m} \int_0^\pi d\theta \sin\theta \, B_{mnn'}(\theta) \frac{d}{dx} [x^2 h_n(x) j_{n'}(x)]_{x=kr(\theta)}$$

$$+ \frac{1}{2\epsilon_m} \int_0^\pi d\theta \sin\theta \, C_{mnn'}(\theta) [x^3 h_n(x) j_{n'}(x) \partial(1/x^2)/\partial\theta]_{x=kr(\theta)} \qquad (42)$$

valid for $n \neq n'$, with

$$B_{mnn'}(\theta) \equiv \frac{m^2 P^m_n \, P^m_{n'}}{\sin^2\theta} + \frac{\partial P^m_n}{\partial\theta} \, \frac{\partial P^m_{n'}}{\partial\theta} \quad , \tag{43a}$$

$$C_{mnn'}(\theta) = n'(n'+1) \frac{\partial P^m_n}{\partial\theta} \, P^m_{n'} + n(n+1) \, P^m_n \frac{\partial P^m_{n'}}{\partial\theta} \quad . \tag{43b}$$

It is convenient this time to write

$$1/x^2 \doteq \text{const.} + \sin^2\theta \quad .$$

Using this in conjunction with the inverse polynomial expression

$$x^2 n_{n+2s} j_n \doteq x^2 n_n j_{n+2s} + \frac{1}{x^{2s-1}} + \frac{1}{x^{2s-3}} + \cdots + \frac{1}{x} \quad ,$$

Eq. (42) may be reduced to

$$J_{m(n+2s)n} - J_{mn(n+2s)}$$

$$\doteq (2s-1)/\epsilon_m \int_0^\pi d\theta \, \sin\theta \, B_{m(n+2s)n} \sum_{q=1}^{s} (\sin\theta)^{2q}$$

$$+ 1/\epsilon_m \int_0^\pi d\theta \, \sin^2\theta \, \cos\theta \, C_{m(n+2s)n} \sum_{q=0}^{s-1} (\sin\theta)^{2q} \quad , \tag{44}$$

where in the first term the constant term in the summation has been dropped because of the additional orthogonality relations

$$\int_0^{\pi} d\theta \sin\theta \, B_{mnn'} = 0, \; n \neq n' \quad .$$

At this point, using the standard recursion formulas for the Legendre functions one can write

$$\sin^2\theta \, B_{m(n+2s)n} \doteq P^m_{n+2s-1} P^m_{n+1} + P^m_{n+2s-1} P^m_{n-1} + P^m_{n+2s} P^m_{n}$$

$$+ P^m_{n+2s+1} P^m_{n-1} + P^m_{n+2s+1} P^m_{n+1} \quad , \tag{45a}$$

$$\sin\theta \cos\theta \, C_{m(n+2s)n} \doteq P^m_{n+2s-1} P^m_{n+1} + P^m_{n+2s-1} P^m_{n-1}$$

$$+ P^m_{n+2s+1} P^m_{n-1} + P^m_{n+2s+1} P^m_{n+1} \quad . \tag{45b}$$

The polynomials in $\sin^2\theta$ appearing in Eq. (44) may be expanded in Legendre polynomials of highest index 2(s-1) [Note that a factor $\sin^2\theta$ has been taken out in the first case to employ in Eq. (45a)]. By examination of Eq. (39) it may be seen because of orthogonality that only the first term on the right-hand side of Eqs. (45a) and (45b)

will make a non-zero contribution to their respective integrals. Writing out these non-vanishing terms in Eq. (44) explicitly, one finally obtains

$$J_{m(n+2s)n} - J_{mn(n+2s)}$$

$$\doteq - \frac{(2s-1)(n+2s+1)(n+2s+m)(n-m+1)}{\epsilon_m (2n+4s+1)(2n+1)} \int_0^{\pi} d\theta \sin\theta \, P^m_{n+2s-1} P^m_{n+1} (\sin\theta)^{2s-2}$$

$$+ \text{ same expression}$$

$$\equiv 0 \quad . \tag{46}$$

Thus I and J are symmetric, and one need only compute elements on and above the diagonal in Eqs. (30). The matrices are expected to be well-conditioned in the sense that numerical results will converge rapidly to final values versus truncation. From the point of view of the Rayleigh expansion in powers of ka, valid at low frequencies, observe that all matrices may be expanded in powers of ka, e.g., writing $(J^{(m)})_{nn'} = J_{mnn'}$, one has

$$J^{(m)} = A + B(ka) + C(ka)^2 + \ldots$$

$$= A\,[1 + A^{-1}(J^{(m)} - A)] \tag{47}$$

where A is diagonal, B is tridiagonal (all elements zero except on, one above, and one below the diagonal) and so forth. The inverse, expanded in powers of ka, is readily obtainable by the binomial theorem as

$$(J^{(m)})^{-1} = [1 - A^{-1} (J^{(m)}-A)+ \dots] A^{-1} \quad . \tag{48}$$

INTERPRETATION OF NUMERICAL RESULTS

In order to provide some insight into the behavior in practice of the various matrices discussed above, the numerical printout for an example has been included (Appendix III). In addition to providing a test case for use with the computer program, many features of matrix behavior are most conveniently described by reference to this printout.

The obstacle to which the results refer consists of a sphere-cone-sphere, as shown in Figure 2. The analytical description of this shape as inputted to the computer is detailed in Paragraph 9.0 of Appendix I. It will be seen that the printout consists almost entirely of matrix quantities, as an aid in gauging the numerical effectiveness of the truncation being employed.

The first page lists input parameters. Thus, four cases (m = 0, 1, 2, 3) were evaluated consecutively, with truncation to 6 x 6 matrices. The body is described analytically in three sections. The body shape "9" indicates that the body does not possess mirror symmetry normal to the axis of rotational symmetry. U vector indicates that 46 aspect

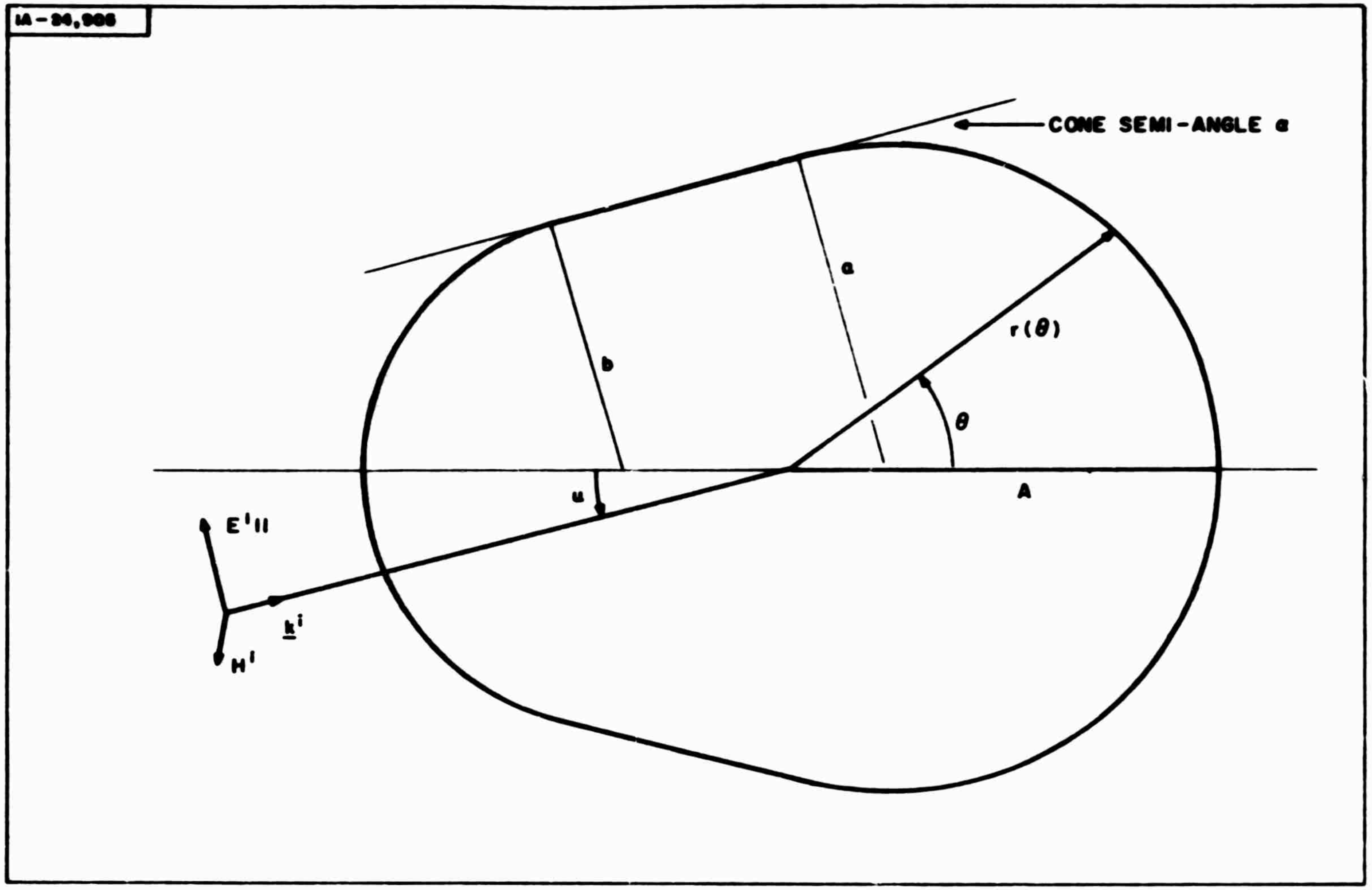

Figure 2. Geometry of the Sphere-Cone-Sphere Body Showing E || Incident Polarization

angles will be evaluated. The body size is $kA = 1.0$ (Figure 2), and the ratio of sphere radii is $b/a = (1 + \sin\alpha)^{-1} \simeq 0.794$ (correct to nine significant figures in the computer), with cone half-angle $\alpha = 15$ degrees. Numerical integration is performed by Bode's rule using 64 equally spaced divisions in each section, and the angular end points in degrees are indicated for each section.[14]

The basic quantities shown, for each m value successively, are the Q matrix, the orthogonalized $\hat{Q}$ matrix, the transition matrix T, and the cumulative cross section quantities.

Consider first the case $m = 0$. The blocks Q_2 and Q_3 are zero for this case, and hence are not shown. The remaining blocks, Q_1 and Q_4, are obtained by numerical integration from the defining equations (16, 29, 30, 31, 32). Because of the vanishing of the blocks Q_2, Q_3 it turns out, as one can verify with some study, that the remaining two blocks actually are processed with no interactions, so that behavior can be discussed by examining say, Q_1, alone.

Considering the imaginary part of Q_1, which is the numerically dominant portion, one observes that elements of each of the six row vectors increase in numerical magnitude as one moves to the right. It is immediately clear that row vectors must be orthogonalized from the bottom up, in order that elements of the resulting vectors may settle down to constant values independent of truncation (that is, if one began orthogonalizing with the top row, then it can be seen that after

normalizing, the first element is smaller by a factor $14.1/11.1 \sim 13$ than it would have been in 5×5 truncation). Observe also that the bottom row is the best candidate for a "unit vector in the six direction," in that the sixth element is larger than the first by a factor of about 10^7, whereas the corresponding factor for the top row is only 2×10^2.

The orthogonalized matrix $\hat{Q}$ unit, Eq. (23), is shown next, having dimension 12 x 12. Odd numbered rows have come from the original 6×6 Q_1, whereas the even numbered rows are associated with Q_4. It is striking to observe that each of the original matrices (and hence the entire imaginary part of $\hat{Q}_{unit}$) has become nearly diagonal. This occurred because the main effect of the sixth row vector was to reduce the last entry of each preceding row to nearly zero. The new fifth row vector, in similar fashion, then served primarily to reduce the fifth entry in each of the preceding four rows, and so on. Thus the first row of Q_1, which originally <u>increased</u> by a factor of about 170 from first to last entry, now <u>decreases</u> by a factor (see row one of the imaginary part of $\hat{Q}_{unit}$) of about 7×10^{-8}. The total relative reduction is of order 4×10^{-10}.

It is not difficult to study the behavior of the row vectors, or the individual matrix elements, versus truncation. For example, if a 5 x 5 truncation had been employed, then the first entry in the fifth row vector would have been, after normalization, $(-1.278 + i0.5861) \times 10^{-5}/(0.5646) = (-2.263 + i1.038) \times 10^{-5}$, whereas the 6 x 6 truncation (see row nine of $\hat{Q}_{unit}$) actually gives $(-2.251 + i1.036) \times 10^{-5}$. One

can verify that the sixth row vector has even less effect on rows earlier than the fifth. In particular, orthogonalizing the first row to the sixth row will change the first element of the first row from 0.848 to approximately 0.848 - (144/0.551) (6.99 x 10^{-8}), which constitutes a change in the sixth significant figure.

The blocks of the transition matrix, computed from Eq. (27) and the reverse transformation of Eq. (18), are shown next. Again, the blocks $T_2 = T_3'$ vanish identically and are not shown. Both T_1 and T_4 are seen to be exactly symmetric to the number of digits given, and the unitary-related condition of Eq. (15) is readily verified on the desk computer to within round-off error in the last place shown. Observe that the elements of both T_1 and T_4 fall off rapidly in magnitude moving away from the upper left hand corner, so that the scattering behavior would be efficiently and accurately describable in this instance using only the first two rows and columns of T_1 and T_4.

Finally, the accumulated (over m = 0 only) far field quantities are shown for $E\parallel$ (class 1) and $E\perp$ (class 2) polarizations. The first column gives the incident aspect angle measured from the axis of rotational symmetry. For each aspect, subsequent columns give the scattering cross section [Eq. (12a)], forward amplitude [the complex quantity appearing in Eq. (12b)], backscattered amplitude [the complex quantity appearing in Eq. (12c) before squaring], and finally the radar cross section and phase of the back scattered amplitude. Observe that

both energy conservation (equality of the second and fourth columns) and reciprocity (symmetry of the third and fourth columns about the aspect angle of ninety degrees) are satisfied to seven significant figures.

Turning to the case $m = 1$, the blocks Q_1 and Q_2 are shown, Q_3 and Q_4 then being given by [see Eqs. (8b, 16, 29)] $Q_3 = Q_2$, $Q_4 = -Q_1 + i\underline{1}$. A partial check on the precision of numerical integration is available for this and all subsequent m values. From the Wronskian relation $x^2[j_n(x)h_{n-1}(x) - j_{n-1}(x)h_n(x)] = i$ and the first of Eqs. (30) it is immediately clear that the imaginary parts of the first off-diagonal elements of Q_2 should be symmetric, a result not used in the program (whereas symmetry of $\text{Re}Q_1$ and $\text{Re}Q_2$ is always enforced). The expected symmetry is seen to obtain to seven significant figures for the (1, 2) and (2, 1) elements. Precision subsequently deteriorates slightly so that discrepancies have appeared in the fifth significant figure between the (5, 6) and (6, 5) elements.

Numerical behavior of both the Q and the orthogonalized $\hat{Q}$ matrices appears to proceed substantially as in the case discussed above for $m = 0$, although the details are of course considerably more complex because of the presence of all four non-zero blocks. The near-diagonal nature of the orthogonalized $\hat{Q}$ matrix is still evident by inspection, however. In the resulting transition matrix the blocks T_1 and T_4 are seen to be symmetric, and the block T_3 to be equal to the transpose of T_2. Comparison of the far field results with those for $m = 0$ reveals significant changes at all aspect angles.

For the two subsequent cases m = 2, 3 a new effect is seen over and above the previously discussed features, due to the vanishing of the associated Legendre functions P_n^m for $n < m$. In consequence, the first row and column of each block of the Q matrix (for m = 3 the first two rows and columns) are identically zero. The behavior versus truncation at n = 6, as judged by the near-diagonal results after orthogonalization, appears unaffected, however. The net result is that the computation becomes gradually simpler as m increases, m = 2 requiring treatment of 5 x 5 blocks, and m = 3 requiring only 4 x 4 blocks.

A measure of error incurred by stopping at m = 3 may be obtained by comparing the far field results with those obtained at m = 2 (except for incidence along the axis of rotational symmetry, 0 degrees or 180 degrees, for which scattering behavior is completely determined from the m = 1 results only). At incidence 80 degrees from the axis, for example, and for either polarization, the scattering cross section is seen to be unchanged to about six significant figures. For the same cases the radar cross section, however, has changed in about the third significant figure. Such precision is nevertheless quite adequate in most practical applications.

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APPENDIX I

ORGANIZATION OF THE COMPUTER PROGRAM

1.0 INTRODUCTION*

The "EMSCAT" Program has been written in FORTRAN IV language for the IBM 7030 Computer to produce solutions to the electromagnetic scattering problems which are outlined above in Section II. Several factors were given consideration in the design of the program:

Efficient coding to reduce computer run time as much as possible. The routine VECMUL for matrix by vector multiplication was coded in machine language to take advantage of specialized coding available at that level. This routine is also available in FORTRAN (though less efficient and accurate) so that the program can be run on machines other than the 7030 Computer.

Full single word accuracy of a 7030 register and where necessary double precision accuracy was utilized in the calculation of special functions. Single precision accuracy on the 7030 maintains 15 digits of accuracy.

Maximum use of core storage. The size of the solution matrices (60 x 60 complex) was determined so that secondary storage devices such as tapes do not have to be utilized in running the program.

The matrices are stored and manipulated from 1 of 3 large blocks of common storage. Each block is dimensioned 120 x 120. However, through various equivalence statements in the proper routines, these major blocks are resegmented and renamed for ease of programming.

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*The paragraphs in Appendix I have been numbered to facilitate cross-referencing.

2.0 GLOSSARY OF THE SUBROUTINES

The program operates via a MAIN routine and 15 auxiliary routines which are briefly described and listed below. Standard I/O and mathematical routines, e.g., SIN, LOG, etc. are assumed to be available through the FORTRAN operating system.

2.1 The MAIN routine controls overall run processing and computes the I, J, K and L matrices.

Routines called are:

RDDATA
GENLGP
GENKR
GENBSL
PRTMTX
PRCSSM

2.2 Subroutine RDDATA reads the user's control parameters and sets up preliminary output heading information.

Routine called is: CALENP

2.3 Subroutine CALENP computes the sections of θ, the polar angle, and the step size for numerical integration.

2.4 Subroutine GENLGP computes the associated Legendre functions over the necessary range.

2.5 Subroutine GENBSL controls backward recursion of Bessel functions and forward recursion of Neumann functions.

2.6 Subroutine BESSEL computes the Bessel function for a specific argument and order.

2.7 Subroutine GENKR computes the parameter "kr" and its derivative with respect to the polar angle θ.

2.8 Subroutine PRTMTX prints the headings and controls the printout of the I, J, K and L complex matrices.

Routine called is: PRINTM

2.9 Subroutine PRINTM prints the elements of a specified matrix of specified rank.

2.10 Subroutine PRCSSM generates the Q matrices from the I, J, K and L matrices, and transforms the Q matrix into the T matrix.

Routines called are: NRMQMX
CNDTNQ
PRTRIT
ADDPRC

2.11 Subroutine NRMQMX normalizes the I, J, K and L matrices to obtain the Q matrices.

2.12 Subroutine CNDTNQ conditions the Q matrix before transforming it into the T matrix.

2.13 Subroutine PRTRIT prints headings and controls printout of the T matrix.

Routine used is: PRINTM

2.14 Subroutine ADDPRC does final processing of the T matrix to provide the scattering results.

Routines called are: GENLGP
VECMUL

2.15 Subroutine VECMUL multiplies a matrix times a vector.

2.16 Subroutine DUMP gives a listing of core storage when an error condition occurs.

Subsequent paragraphs detail the above routines where necessary. It should be noted at this point that standard mathematical notation is not necessarily followed, e.g., program notation labels Bessel functions as B instead of j. This was done for ease of relating program mnemonics to mathematical notation. When necessary, parameters have been labeled which have notation different from the earlier text.

3.0 THE INPUT ROUTINE

Subroutine RDDATA reads the user's control information, prints out headings and obtains information for numerical integration. The input cards and their formats are listed below.

3.1 Card 1 — NM, NRANK, NSECT, IBODY, NUANG

FORMAT (5I12)

NM	No. of values of "m". See Card 3.
NRANK	Rank of matrices I, J, K and L
NSECT	No. of sections defining body shape and integration intervals. See Subroutine CALENP for fuller description of body shapes.
IBODY	Case No. or body shape identifier 7 : Spheroid 8 : Mirror Symmetry 9 : General Axisymmetric Case
NUANG	No. of aspect angles "u". See Card 5.

3.2 Card 2. CONK, BRXT, ALPHA

FORMAT (3E12.7)

CONK — ka, scale factor for r, the polar radius, in determining body shape.

BRXT — variable parameter to be used in computing body shapes.

ALPHA — α, or a variable parameter, to be used in calculating body shapes. For a fuller description of its usage see Subroutines CALENP and GENKR described below.

3.3 Card(s) 3. CMI(I), I = 1, NM

FORMAT (6E12.7)

CMI(I) — I^{th} value of "m" to be used in current solution of scattering problem. As many as 30 values of "m", the azimuthal index, may be read in; "m" is any integer ≥ 0.

3.4 Card 4. NDPS(I), I = 1, NSECT

FORMAT (6I12)

NDPS(I) — No. of divisions for integration in I^{th} section of the body shape. The body may be divided into as many as 6 sections. These parameters are used to calculate spacing for numerical integration, and they must be a multiple of 4.

3.5 Card(s) 5. UANG(I), I = 1, NUANG

FORMAT (6I12)

UANG(I) — I^{th} value of "u", a member of a table of aspect angles (in degrees). As many as 60 values of "u" may be read in.

4.0 CALCULATION OF END POINTS AND SPACING FOR INTEGRATION

Subroutine CALENP is one of two special routines that have to be written into the program for specific body shapes. This routine calculates "NSECT" values of the polar angle θ, which provid: boundaries for dividing the body into sections for numerical integration. With each boundary point a value of $\dot{\theta}$ is associated. The spacing for integration is then determined by dividing the range of θ by the correct value of "NDPS". Note that the number of divisions does not have to remain constant from one section to the next, but it must be a multiple of 4.

Since the computations for each special version of CALENP may vary, the following parameters, "ALPHA", "BRXT", "QB", "SNALPH", and "CSALPH" may be used for communicating between routines special values associated with a particular body shape. Note the use of QB below for a variable peculiar to the sphere- ne-sphere shape.

In Appendix II, a listing of the routines CALENP and GENKR are given for a sphere-cone-sphere body (Figure 2.).

For the sphere-cone-sphere body, three end points for θ; θ_1, θ_2, θ_3, are computed as follows:

$$\theta_1 = \tan^{-1}\left[\frac{\sin\alpha\,\cos\alpha}{q - \sin^2\alpha}\right];\ 0 \le \theta_1 \le 105^\circ$$

NOTE: α (Figure 2) is an input parameter stored at "ALPHA"

b/a (Figure 2) is an input parameter stored at "BRXT"

$$q = \frac{(1 - b/a)(1 - \sin\alpha)}{2}$$

and is computed and stored in QB.

$$\theta_2 = \tan^{-1}\left[\frac{(b/a)\sin\alpha\cos\alpha}{1-q-(b/a)\cos^2\alpha}\right] ; \theta_1 < \theta_2 < \pi$$

$$\theta_3 = \pi$$

5.0 THE FIRST CONTROL ROUTINE

The MAIN Routine controls the general flow of the program and computes the real and imaginary parts of the partitioned scattering matrices, I, J, K and L. After the user's control data has been read in, a numerical integration system utilizing Bode's 3rd order rule is utilized.(14) The program computes the I and J complex matrix elements for the three cases, axisymmetric, mirror-symmetric and spheroidal as follows:

5.1

$$I_{ij} = m\int_0^{\pi} (\sin\theta)\,(kr)^2\, B_j(kr)\, H_i(kr) \Big\{(i + j)\cos\theta\, P_i^m P_j^m$$

$$- (i + m)\, P_j^m P_{i-1}^m \quad - \quad (j + m)\, P_i^m P_{j-1}^m\Big\}\, d\theta$$

$B_j(kr)$: Bessel function of the first kind of order "j" and argument "kr"

$H_i(kr)$: Hankel functions which are defined as

$$B_i(kr) + \sqrt{-1}\, N_i(kr)$$

$N_i(kr)$: Neumann functions of order "i" and argument "kr".

$$P_i^m = \frac{P_i^m (\cos\theta)}{\sin\theta}$$

where

$$P_i^m (\cos\theta)$$

is the associated Legendre function, of rank m, and order i.

r : Polar radius used in calculating body shape.

i : Subscript notation for i^{th} row of the matrix,

j : Subscript notation for j^{th} column of the matrix.

5.2

$$J_{ij} = \frac{-2}{e_m} \int_0^{\pi} (\sin\theta)\, H_i(kr) \Big\{ P_i^m P_j^m \Big[kr(kr\, B_{j-1}(kr) - jB_j(kr)\, (m^2 + i\, j\cos^2\theta)$$

$$+ i\, j(j + 1) \frac{d(kr)}{d\theta} B_j(kr) \sin\theta \cos\theta \Big]$$

$$-(i + m) P_{i-1}^m P_j^m \Big[kr \cos\theta (kr\, B_{j-1}(kr) - j\, B_j(kr) + (j+1) \frac{d(kr)}{d\theta} B_j(kr) \sin\theta \Big]$$

$$+(j+m)P^m_{j-1}\ kr\left[\left(kr\ B_{j-1}(kr)-j\ B_j(kr)\right)\left((i+m)P^m_{i-1}-i\cos\theta\ P^m_i\right)\right]\Bigg\}d\theta$$

where:

$$e_m = \begin{cases} 1 & \text{for } m = 0 \\ 2 & \text{for } m \geq 1 \end{cases} .$$

Within the program each element of the I and J arrays is used as an accumulator for numerical integration under Bode's rule. Thus, for a specified value of θ, all necessary functions are computed and added to the correct matrix element.

To save computer time, computations which would produce a null contribution to the integration are eliminated, and the following symmetries are taken advantage of in the direct computation of the I and J matrices.

5.3 General axisymmetric bodies:

$$Re(I_{ji}) = Re(I_{ij})$$

$$Re(J_{ji}) = Re(J_{ij})$$

5.4 Mirror-symmetric bodies: use paragraph 5.3 plus

$$I_{ij} = 0; \text{ if } i \text{ and } j \text{ are} \begin{cases} \text{both odd} \\ \text{both even} \end{cases}$$

$$J_{ij} = 0; \text{ if } i \text{ and } j \text{ are} \begin{cases} \text{odd, even} \\ \text{even, odd} \end{cases}$$

5.5 Spheroids: use paragraphs 5.3, 5.4, plus

$$Im(I_{ji}) = Im(I_{ij})$$

$$Im(J_{ji}) = Im(J_{ij})$$

The K and L matrices are then calculated from the following relationships with the I and J matrices:

5.6 $$Re(K_{ij}) = -Re(J_{ij})$$

$$Im(K_{ij}) = -Im(K_{ij}) + b\, D_{ij}$$

where

$$b = \begin{cases} 1.0 & \text{General Axisymmetric bodies} \\ 0.5 & \text{Mirror-Symmetric or Spheroidal bodies} \end{cases}$$

$$D_{ij} = \begin{cases} 0.0; & i \neq j \\ \left[\dfrac{\epsilon_m (2i+1)}{4i(i+1)} \dfrac{(i-m)!}{(i+m)!}\right]^{-1} & ; i = j \end{cases}$$

5.7 $$Re(L_{ij}) = -Re(I_{ij})$$

$$Im(L_{ij}) = -Im(I_{ij})$$

The I, J, K and L complex matrices are then printed by Subroutine PRTMTX and control passes to Subroutine PRCSSM for further processing.

6.0 ASSOCIATED LEGENDRE FUNCTIONS

Subroutine GENLGP generates the associated Legendre functions, $P_i^m(x)$ for a given argument x, a given value of the azimuthal index m,

and for all values of degree i from 0 to "NRANK", the input-specified rank of the matrix. The first two values of P are generated by formula, then the remaining values of P are generated by a recursion relationship.

The following formulae are used to generate $P_i^m(x)$. Note that for this particular program, the functions always appear in the context

$$\frac{P_i^m(\cos\theta)}{\sin\theta} .$$

$$\frac{P_i^m(\cos\theta)}{\sin\theta} = 0.0 \qquad ; \quad i < m$$

$$\frac{P_m^m(\cos\theta)}{\sin\theta} = \frac{(2m)!\ \sin^{m-1}(\theta)}{2^m \cdot m!} \qquad ; \quad i = m$$

$$\frac{P_0^0(\cos\theta)}{\sin\theta} = \frac{1.0}{\sin\theta} \qquad ; \quad i = m = 0$$

$$\frac{P_1^0(\cos\theta)}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

Recursion relationship:

$$\frac{P_n^m(\cos\theta)}{\sin\theta} = \frac{(2n-1)\cos\theta \left[\frac{P_{n-1}^m(\cos\theta)}{\sin\theta}\right] - (n + m - 1)\left[\frac{P_{n-2}^m(\cos\theta)}{\sin\theta}\right]}{n - m}$$

7.0 BESSEL FUNCTIONS

Subroutine BESSEL generates a Bessel function of the first kind $B_n(x)$, for a specified argument x, and order n, by means of an infinite series. To preserve accuracy, the computations are performed in double precision arithmetic and truncation error due to neglected terms in the series is $< 10^{-20}$. If the series has not converged to the aforementioned accuracy beforethe computation of the 100^{th} term, an error indication is given.

The following infinite series is used to compute a Bessel function:

$$B_n(x) = \frac{x^n}{1\cdot 3\cdot 5\cdots(2n+1)} \sum_{i=0}^{\infty} a_i$$

where:

$$a_0 = 1.0$$

$$a_i = \frac{-x^2}{2i\left[2n + (2i+1)\right]}\, a_{i-1}$$

8.0 RECURSION RELATIONSHIPS FOR BESSEL AND NEUMANN FUNCTIONS

Subroutine GENBSL calls Subroutine BESSEL to obtain two successive BESSEL functions for a specified argument, and then uses these first two values to recurse backward over the range of i from NRANK to 0. If the two computed functions of order NRANK and NRANK-1 do not satisfy the accuracy requirements mentioned in paragraph 7.0,

the routine will increase the order of the computed BESSEL function to 4(NRANK). If this fails to produce a satisfactory pair of functions, the run will abort and a dump of core memory is taken.

The recursion relation used for computing BESSEL functions is:

$$B_{n-1}(x) = (2n + 1)x^{-1} B_n(x) - B_{n+1}(x) \quad .$$

This routine also computes Neumann functions by a forward recursion formula after the first two values are computed by the following formulae:

$$N_0(x) = \frac{-\cos x}{x}$$

$$N_1(x) = \frac{-\cos x}{x^2} - \frac{\sin x}{x} \quad .$$

The recursion relation used for computing the remaining Neumann functions is:

$$N_{n+1}(x) = (2n+1)x^{-1} N_n(x) - N_{n-1}(x) \quad .$$

To test the accuracy of the functions over the range of computed Bessel and Neumann functions for a given argument, two tests are performed in the <u>MAIN</u> Routine after the vector of functions from 0 to <u>NRANK</u> is computed. If the following relations are not satisfied to an accuracy of 10^{-10}, an error message indicating such a condition is printed, and the program continues. Though the tests are performed

in the MAIN Routine after the call to Subroutine GENBSL, for convenience they are listed here:

Bessel Test:

$$\left| x^2 \left[B_1(x)N_0(x) - B_0(x)N_1(x) \right] - 1 \right| < 10^{-10} \quad .$$

Neumann Test:

$$\left| x^2 \left[B_{NRANK}(x)N_{NRANK-1}(x) - B_{NRANK-1}(x)N_{NRANK}(x) \right] - 1 \right| < 10^{-10} \quad .$$

9.0 GENERATING THE BODY SHAPE

Subroutine GENKR is one of two custom written routines which are adapted to the particular body shape in question. As noted above in Subroutine CALENP certain parameters are available to the programmer to use as he sees fit to communicate information from one routine to another. The basic function of all versions of GENKR is to compute the polar radius r as a function of the polar angle θ, to compute $\frac{dr}{d\theta}$ and to scale these values by the input constant ka = CONK.

To illustrate the use of this routine, a sphere-cone-sphere body shape is used (Figure 2). As a result of subroutine CALENP the major divisions of the body as a function of θ have been recorded. This routine, given a value of θ now computes $(ka)r$ and $ka(dr/d\theta)$; the scale factor ka is an input to the program.

9.1 Section 1 $0 \le \theta \le \theta_1$

$$r = \frac{q \cos\theta}{\sin\alpha} + \left[1 - \left(\frac{q \sin\theta}{\sin\alpha} \right)^2 \right]^{\frac{1}{2}}$$

$$\frac{dr}{d\theta} = \frac{-q \sin\theta}{\sin\alpha} - \left(\frac{q}{\sin\alpha}\right)^2 \sin\theta \cos\theta \left[1 - \left(\frac{q \sin\theta}{\sin\alpha}\right)^2\right]$$

NOTE: q was computed in Subroutine CALENP and stored in location QB.

9.2 Section 2 $\theta_1 < \theta \leq \theta_2$

$$r = \frac{1 - q}{\sin(\theta-\alpha)}$$

$$\frac{dr}{d\theta} = \frac{-(1-q)\cos(\theta-\alpha)}{\sin^2(\theta-\alpha)}$$

9.3 Section 3 $\theta_2 < \theta \leq \pi$

$$r = -\left[\frac{1-(b/a)-q}{\sin\alpha}\right]\cos\theta + \left[(b/a)^2 - \left(\frac{1-(b/a)-q}{\sin\alpha}\right)^2 \sin^2\theta\right]^{\frac{1}{2}}$$

$$\frac{dr}{d\theta} = \left[\frac{1-(b/a)-q}{\sin\alpha}\right]\sin\theta - \left(\frac{1-(b/a)-q}{\sin\alpha}\right)^2 \sin\theta \cos\theta \left[(b/a)^2 - \left(\frac{1-(b/a)-q}{\sin\alpha}\right)^2 \sin^2\theta\right]^{\frac{1}{2}}$$

10.0 FIRST MATRIX PRINTOUT

Subroutine PRTMTX controls the printout of the I, J, K and L matrices. Both the real and imaginary arrays comprising each of these matrices are labeled and printed out on the community output tape. This output, which was originally intended as an intermediate printout for checking the program, may be eliminated by removing the "CALL PRTMTX"

statement which follows Fortran statement 860 in the MAIN Routine.

11.0 PRINTOUT OF AN ARRAY

Subroutine PRINTM will print out a specified square array of given rank.

12.0 GENERATING THE Q MATRIX AND THE T MATRIX

Subroutine PRCSSM is the second major control routine and it controls the transformation of the I, J, K and L matrices to the "Q" matrices, and the subsequent solution of a matrix equation which provides the "T" matrix.

Subroutine NRMQMX (see below) normalizes the I, J, K and L matrices to produce the Q matrix.

For notational convenience we define:

$$Q = Re(Q) + i\, Im(Q) = \begin{pmatrix} Q_1 & Q_2 \\ Q_2 & Q_4 \end{pmatrix}$$

where

$$i = \sqrt{-1} .$$

The method currently used by the program to transform the Q matrix into the T matrix involves orthogonalizing the Q matrices. After the complex Q matrix has been generated by normalizing the I, J, K and L matrices it is in the form noted in paragraph 12. From these Q matrices, a new complex $\hat{Q}$ matrix of rank 2N is generate from the following relations:

$$\hat{Q}_{(2m-1)\ (2n-1)} = (Q_1)_{m\ n}$$

$$\hat{Q}_{(2m-1)\ (2n)} = (Q_2)_{m\ n}$$

$$m,n = 1,\ 2,\ \ldots.,\ N$$

$$\hat{Q}_{(2m)\ (2n-1)} = (Q_3)_{m\ n}$$

$$\hat{Q}_{(2m)\ (2n)} = (Q_4)_{m\ n}$$

The new $\hat{Q}$ matrix is next conditioned as outlined in Subroutine <u>CNDTNQ</u> of paragraph 14.0 below.

Orthogonalization then proceeds as follows.

1) Consider each row of $\hat{Q}$ as a vector with 2N components; e.g. the components of the first vector $\underline{Q}_1$ would be:

$$\underline{Q}_{1\ 1},\ \underline{Q}_{1\ 2},\ \underline{Q}_{1\ 3},\ \ldots.\ \underline{Q}_{1(2N)}$$

Orthogonalization will proceed from the bottom or $2N^{th}$ vector upward.

2) Normalize the $2N^{th}$ vector as follows:

$$\underline{Q}_{2N} = \frac{\underline{Q}_{2N}}{(\underline{Q}^*_{2N} \cdot \underline{Q}_{2N})^{\frac{1}{2}}}$$

where the scalar product of the complex conjugate $\underline{Q}^*_p$ by another vector $\underline{Q}_q$ is defined as follows:

$$\underline{Q}_p^* \cdot \underline{Q}_q = \sum_{r=1}^{2N} \underline{Q}_{pr}^* \underline{Q}_{qr} = \underline{Q}_{p1}^* \underline{Q}_{q1} + \underline{Q}_{p2}^* \underline{Q}_{q2} + \cdots\cdots \underline{Q}_{p(2N)}^* \underline{Q}_{q(2N)}$$

3) Orthogonalize $\hat{\underline{Q}}_{(2N-1)}$ to $\hat{\underline{Q}}_{(2N)}$:

$$\hat{\underline{Q}}_{2N-1} = \hat{\underline{Q}}_{2N-1} - \left[\hat{\underline{Q}}_{2N}^* \cdot \hat{\underline{Q}}_{2N-1}\right]\hat{\underline{Q}}_{2N}$$

4) Normalize $\hat{\underline{Q}}_{2N-1}$:

$$\hat{\underline{Q}}_{2N-1} = \frac{\hat{\underline{Q}}_{2N-1}}{\left[\underline{Q}_{2N-1}^* \cdot \underline{Q}_{2N-1}\right]^{\frac{1}{2}}}$$

5) Orthogonalize $\hat{\underline{Q}}_{2N-2}$ to both $\hat{\underline{Q}}_{2N}$ and $\hat{\underline{Q}}_{2N-1}$:

$$\hat{\underline{Q}}_{2N-2} = \hat{\underline{Q}}_{2N-2} - \left[\hat{\underline{Q}}_{2N-1}^* \cdot \hat{\underline{Q}}_{2N-2}\right]\hat{\underline{Q}}_{2N-1} - \left[\hat{\underline{Q}}_{2N}^* \cdot \hat{\underline{Q}}_{2N-2}\right]\hat{\underline{Q}}_{2N}$$

6) Normalize $\hat{\underline{Q}}_{2N-2}$:

$$\hat{\underline{Q}}_{2N-2} = \frac{\hat{\underline{Q}}_{2N-2}}{\left[\hat{\underline{Q}}_{2N-2}^* \cdot \hat{\underline{Q}}_{2N-2}\right]^{\frac{1}{2}}}$$

7) Continue the orthogonalization and normalization process until $\hat{\underline{Q}}_1$ has been orthogonalized to all subsequent rows.

8) A complex matrix $\hat{T}$ is now generated from the complex matrix $\hat{Q}$ by the following relation:

$$\hat{T} = \hat{Q}^{*}\mathrm{Re}(\hat{Q})$$

9) The $\hat{T}$ matrix is then decomposed into the matrices T_1, T_2, T_3 and T_4 by the reverse of the procedure in paragraph 12.1.

The complex T matrix is printed by Subroutine <u>PRTRIT</u> and then the final processing is performed by Subroutine <u>ADDPRC</u>.

13.0 NORMALIZINC MATRICES

The Subroutine <u>NRMQMX</u> normalizes the I, J, K and L matrices to obtain the Q matrix. The Q matrix is blocked as noted above in paragraph 12.0 and the following procedure is used:

$$Q_1 = (Z2)^{-\frac{1}{2}} J' (Z2^{-\frac{1}{2}}) \equiv Q_{ij} = \frac{J_{ij}}{\sqrt{Z2_i} \cdot \sqrt{Z2_j}}$$

$$Q_2 = -(Z2)^{-\frac{1}{2}} L' (Z2)^{-\frac{1}{2}}$$

$$Q_3 = (Z2)^{-\frac{1}{2}} I' (Z2)^{-\frac{1}{2}}$$

$$Q_4 = (Z2)^{-\frac{1}{2}} K' (Z2)^{-\frac{1}{2}}$$

where:

$$Z2_n = \left[\frac{e_m(2n+1)}{4n(n+1)} \frac{(n-m)!}{(n+m)!} \right]^{-1}$$

The expression for $Z2_n$ is the same as that used in computing the K matrix of paragraph 5.6. The prime on J, etc. denotes matrix transpose as seen from the second half of the equality statement.

14.0 CONDITIONING MATRICES

After the matrix $\hat{Q}$ of rank 2N has been formed, the matrix is conditioned starting with the last row $\hat{\underline{Q}}_{2N}$ and working towards row $\hat{\underline{Q}}_1$.

14.1

$$\left(\hat{\underline{Q}}_{2N}\right)_i = \left[1/\mathrm{Im}(\hat{\underline{Q}}_{2N})_{2N}\right]\left(\hat{\underline{Q}}_{2N}\right)_i \quad ; \quad i = 1, 2, \ldots 2N$$

The notation $(\hat{\underline{Q}}_{2N})_i$ refers to the i^{th} element of the $(2N)^{th}$ (last) row vector. Now set

$$\hat{\underline{Q}}_m = \hat{\underline{Q}}_m - \left[\mathrm{Im}(\hat{\underline{Q}}_m)_{2N}\right]\hat{\underline{Q}}_N$$

where the equivalence is performed for each of the 2N elements of $\hat{\underline{Q}}_m$, and repeated for all rows $m = 1, 2, \ldots, 2N-1$.

14.2 Redefine

$$\hat{\underline{Q}}_{2N-1} = \left[1/\mathrm{Im}(\hat{\underline{Q}}_{2N-1})_{2N-1}\right]\hat{\underline{Q}}_{2N-1} ,$$

then compute

$$\hat{Q}_m = \hat{Q}_m - \left[\mathrm{Im}(\hat{Q}_m)_{2N-1}\right]\hat{Q}_{2N-1}$$

for all rows $m = 1, 2, \ldots 2N-2$.

14.3 Continue the process of paragraphs 14.1 and 14.2 for all the remaining rows. The final step in the process is to generate

$$\hat{Q}_2 = \left[1/\mathrm{Im}(\hat{Q}_2)_2\right]\hat{Q}_2 \quad ,$$

$$\hat{Q}_1 = Q_1 - \left[\mathrm{Im}(\hat{Q}_1)_2\right]\hat{Q}_2 \quad .$$

14.4 Set

$$\mathrm{Im}(\hat{Q}_m)_i = 0.0; \quad i = m + 1, m + 2, \ldots, 2N; \quad m = 1, 2, 3, \ldots, 2N-1 \; .$$

15.0 PRINTING THE T MATRIX

Subroutine PRTRIT controls the printout of the T matrix, both real and imaginary elements, in the same manner as Subroutine PRTMTX (paragraph 11.0 above) controls the printout of the I, J, K and L matrices. The community output tape is used. Since this printout is used mainly for checkout, it can be eliminated by removing the "CALL PRTRIT" statement following FORTRAN statement 140 in Subroutine PRCSSM.

16.0 FINAL CONTROL ROUTINE

Subroutine ADDPRC is the third and last control routine which converts the T matrix to the final set of results. Two sets of results are generated, a set of answers for the current value of m and an accumulated set of answers for all values of m up to and including the present value of m.

To generate the final results, the following procedure is followed: The T matrix is normalized

$$T(I.) = (Z2_i)^{\frac{1}{2}} T_{ij}(k) (Z2_j)^{-\frac{1}{2}} \qquad \text{block}$$

where: k indicates 1 of 4 blocks;

NOTE:

$$T = \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix}$$

$Z2_n$ is as defined in Subroutine NRMQMX under paragraph 13.0.

NOTE: For the reader who is relating the mathematics to the program listing in Appendix II, the mapping of COMMON storage in paragraph 19.0 should be consulted.

The associated Legendre functions of form

$$\frac{P_n^m(\cos u)}{\sin u}$$

are generated for each value of the aspect angle u, and for $n = 1$ to NRANK. The derivatives of the Legendre functions are computed from:

$$\frac{d\left[P_n^m(\cos u)\right]}{du} = n \cos u \left[\frac{P_n^m(\cos u)}{\sin u}\right] - (n + m)\left[\frac{P_{n-1}^m(\cos u)}{\sin u}\right].$$

Values of the vectors $\overline{F^1, G^1}$ and $\overline{F^2, G^2}$ are generated by Subroutine VECMUL. These vectors are defined as:

16.1

$$\begin{pmatrix} F^1 \\ \\ G^1 \end{pmatrix} = -i \begin{pmatrix} T_1 & T_2 \\ \\ T_3 & T_4 \end{pmatrix} \begin{pmatrix} (i)^n \, m \, P_n^m(\cos u)/\sin u \\ \\ -(i)^{n+1} \, d\left[P_n^m(\cos u)\right]/du \end{pmatrix}$$

and

16.2

$$\begin{pmatrix} F^2 \\ \\ G^2 \end{pmatrix} = i \begin{pmatrix} T_1 & T_2 \\ \\ T_3 & T_4 \end{pmatrix} \begin{pmatrix} (i)^{n+1} \, d\left[P_n^m(\cos u)\right]/du \\ \\ (i)^n \, m \, P_n^m(\cos u)/\sin u \end{pmatrix}$$

The final sets of answers are generated from the following equations:

$$\text{SCATT 1, 2} = \frac{16}{(ka)^2} \sum_{n=1}^{\text{NRANK}} (Z2)^{-1} \left[|F_n^{1,2}|^2 + |G_n^{1,2}|^2\right]$$

$$\text{TOTAL } 1,2 = \frac{16}{(ka)^2} \sum_{n=1}^{NRANK} (Z2)^{-1}(-i)^n \left[F_n^{1,2} \frac{m\, P_n^m(\cos u)}{\sin u} + iG_n^{1,2} \frac{d\left(P_n^m(\cos u)\right)}{du} \right]$$

$$\text{RTRAD } 1,2 = \frac{8}{(ka)} \sum_{n=1}^{NRANK} (Z2)^{-1}(i)^n \left[F_n^{1,2} \frac{m\, P_n^m(\cos u)}{\sin u} - iG_n^{1,2} \frac{d\left(P_n^m(\cos u)\right)}{du} \right]$$

The final results are divided into two classes as noted above by the quantities SCATT 1,2 etc. The classes are two different incident polarizations. Class 1 is the E-parallel incidence; class 2 is the E-perpendicular incidence.

Appendix III contains a listing of a sample output of the sphere-cone-sphere-body shape. The printout titles and their meanings are:

ANGLE	u, the aspect angle (degrees)
SCATT 1, 2	Scattering cross-section for each class, normalized by $[\pi a^2]^{-1}$
TOTAL 1, 2	Complex forward amplitude, normalized by $[\pi a^2]^{-\frac{1}{2}}$
RTRAD 1, 2	Complex back scattered amplitude normalized by $[\pi a^2]^{-\frac{1}{2}}$
RCS 1, 2	Radar cross section normalized by $[\pi a^2]^{-1}$ NOTE: RCS $\equiv$ $\lvert \text{RTRAD} \rvert^2$ only is computed for the accumulative case.

PHASE ANGLE 1, 2 For the accumulative case, a phase angle is computed:

$$PHANG = TAN^{-1}\left[\frac{Im(RTRAD)}{Re(RTRAD)}\right]$$

This is the phase angle of the back scattered amplitude (Degrees).

17.0 MULTIPLYING A MATRIX TIMES A VECTOR

Subroutine VECMUL is one of two routines coded in both machine language and FORTRAN. It multiplies a matrix times a vector to compute the vectors $\overline{F^1, G^1}$ and $\overline{F^2, G^2}$ of paragraphs 16.1 and 16.2. The machine coded version has the advantages of higher speed and accuracy.

18.0 CORE DUMP

If an abnormal or uncorrectable error condition occurs, Subroutine DUMP gives a dump of core memory as an aid in debugging the error condition. The Subroutine LBPDMP is a system routine for dumping core between specified limits.

19.0 STORAGE ARRANGEMENTS

To conserve and fully utilize core storage, three large matrix arrays of dimension 120 x 120 have been set up in an area of COMMON storage named "MTXCOM". To aid in programming, various routines use EQUIVALENCE statements to resegment these large arrays into manageable blocks.

The FORTRAN array names of the three major blocks are:

CMTXRL (120, 120)
CMTXIM (120, 120)
SPRMTX (120, 120)

Within the MAIN Routine the following overlays are made:

CMTXRL	AMXIR (60, 60)	:	RE(I)
	AMXJR (60, 60)	:	RE(J)
	AMXKR (60, 60)	:	RE(K)
	AMXLR (60, 60)	:	RE(L)
CMTXIM	AMXII (60, 60)	:	IM(I)
	AMXJI (60, 60)	:	IM(J)
	AMXKI (60, 60)	:	IM(K)
	AMXLI (60, 60)	:	IM(L)

The SPRMTX block is unused.

Subroutine NRMQMX and Subroutine PRCSSM, the second control routine, allocated storage as follows:

CMTXRL	QMTXII (60, 60)	:	QI1 (60, 60)	:	IM(Q_1)
	QMTXJI (60, 60)	:	QI2 (60, 60)	:	IM(Q_2)
	QMTXKI (60, 60)	:	QI3 (60, 60)	:	IM(Q_3)
	QMTXLI (60, 60)	:	QI4 (60, 60)	:	IM(Q_4)
SPRMTX	QMTXIR (60, 60)	:	QE1 (60, 60)	:	RE(Q_1)
	QMTXJR (60, 60)	:	QR2 (60, 60)	:	RE(Q_2)
	QMTXKR (60, 60)	:	QR3 (60, 60)	:	RE(Q_3)
	QMTXLR (60, 60)	:	QR4 (60, 60)	:	RE(Q_4)

NOTE: After Subroutine NRMQMX normalizes and moves the Q matrix (complex) into the SPRMTX and CMTXRL areas, the processing which transforms the Q to the T matrix follows the procedure outlined in paragraph 12.0. The storage allocation is noted above in the eight itemized steps.

Subroutine INVMBL always assumes the block matrix which is to be processed is stored in the CMTXRL area. The intermediate steps as outlined in paragraph 14.0 are performed in the CMTXIM area.

The third and last control routine, Subroutine ADDPRC makes the following storage allocations:

CMTXRL	QMTXII (60, 60)	:	FGVECT (2, 120, 2)
	QMTXJI (60, 60)	:	FGMUL (120, 2)
	QMTXKI (60, 60)	:	FGANS (60, 10)
	QMTXLI (60, 60)	:	(unused)

CMTXIM SPRMTX	TCMPLX (2, 120, 120)	:	Real and imaginary components of the T matrix.

NOTE: That TCMPLX overlays both the CMTXIM and SPRMTX areas. As noted in paragraph 16.0, FGVECT contains the $\overline{F^1, G^1}$ and $\overline{G^2, F^2}$ vectors. The first subscript refers to the real and imaginary components of the vectors, the second subscript refers to the dimension of the vectors which is 2·NRANK and the last subscript differentiates the 2 vectors.

FGMUL contains the vectors which post-multiply the T matrix to generate FGVECT. The first and second subscripts correspond to

the second and third subscripts of FGVECT.

FGANS contains the final answers. The first subscript corresponds with the value of aspect angle which generated it and the second subscript refers to the answers in the following manner:

1	2	3	4	5
$SCATT^1$	$Re(TOTAL^1)$	$Im(TOTAL^1)$	$Re(RTRAD^1)$	$Im(RTRAD^1)$

6	7	8	9	10
$SCATT^2$	$Re(TOTAL^2)$	$Im(TOTAL^2)$	$Re(RTRAD^2)$	$Im(RTRAD^2)$

APPENDIX II

THE FORTRAN IV PROGRAM LISTING

```
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C     SCATTERING FROM AXISYMMETRIC CONDUCTORS FOR CASES 7, 8 AND 9.
      COMMON DTR,RTD,CPI
      COMMON /CMVCOM/ NM,CMI(30),CMV,XMV,CM2,EM,QEM,TWM,PRODM
      COMMON /FNCCOM/ PNMLLG(61),BSSLSP(61),CNEUMN(61)
      COMMON /THTCOM/ THETA,NTHETA,DLTHTA,SINTH,COSTH,ISMRL,ISWTCH(7),SR
     1MUL,SMULSS(7),CDH(6),DHM,NSECT,NDPS(6),EPPS(6),KSECT
      COMMON /MTXCOM/ NRANK,NRANKI,AMXIR(60,60),AMXJR(60,60),AMXKR(60,60
     1),AMXLR(60,60),AMXII(60,60),AMXJI(60,60),AMXKI(60,60),AMXLI(60,60)
     2,SPRMTX(120,120),CMXNRM(60)
      COMMON /BCYCOM/ CKR,DCKR,CKR2,CSKRX,SNKRX,CONK,BRXT,ALPHA,IBODY,QB
     1,SNALPH,CSALPH
      DIMENSION CLRMTX(43200)
      EQUIVALENCE (AMXIR,CLRMTX)
C     SET PROGRAM CONSTANTS.
      DTR = 1.7453292519943E-02
      RTD = 57.2957795131
      CPI = 3.1415926535898
      ISWTCH(1) = 2
      ISWTCH(2) = 3
      ISWTCH(3) = 4
      ISWTCH(4) = 1
      SMULSS(1) = 32.0
      SMULSS(2) = 12.0
      SMULSS(3) = 32.0
      SMULSS(4) = 14.0
C     CALL ROUTINE TO READ DATA AND PRINT HEADINGS FOR OUTPUT
   20 CALL RDDATA
      IF(IBODY-9)24,22,24
   22 BDYFCT = 1.0
      GO TO 26
   24 BDYFCT = 0.5
C     SET UP A LOOP FOR M AND SET VARIABLES WHICH ARE A FUNCTION OF M.
   26 DO 900 IM = 1,NM
      CMV = CMI(IM)
      KMV = CMV
      CM2 = CMV*CMV
      PRODM = 1.0
      IF(CMV)40,40,44
   40 EM = 1.0
      GO TO 60
   44 EM = 2.0
      QUANM = CMV
      DO 52 IFCT = 1,KMV
      QUANM = QUANM&1.0
      PRODM = QUANM*PRODM/2.0
   52 CONTINUE
   60 QEM = -2.0/EM
      TWM = CMV&CMV
C     INITIALIZE ALL MATRIX AREAS TO ZERO
      DO 80 I = 1,28800
      CLRMTX(I) = 0.0
   80 CONTINUE
C     SET UP A LOOP FOR ALL VALUES OF THETA.
      THETA = 0.0
C     SET UP GENERAL LOOP FOR CORRECT NUMBER OF INTEGRATION SECTIONS.
      DO 800 ISECT = 1,NSECT
      KSECT = ISECT
      NTHETA = NDPS(ISECT)&1
      DLTHTA = CDH(ISECT)
```

```
      DHM = DLTHTA/22.5
      ISMRL = 4
      DO 700 ITHTA = 1,NTHETA
C     SET SWITCHES AND MULTIPLIERS FOR SIMPSONS INTEGRATION METHOD.
      IF(ITHTA-1)120,120,132
  120 SRMUL = 7.0*DHM
      IF(ISECT-1)700,700,362
  132 IF(ITHTA-NTHETA)200,148,148
  148 SRMUL = 7.0*DHM
      GO TO 340
  200 ISMRL = ISWTCH(ISMRL)
      SRMUL = SMULSS(ISMRL)*DHM
  340 THETA = THETA&DLTHTA
  348 COSTH = COS(THETA)
      SINTH = SIN(THETA)
C     GENERATE THE LEGENDRE POLYNOMIALS.
      CALL GENLGP
C     EVALUATE KR AS A FUNCTION OF THETA. ALSO ITS DERIVATIVE.
      CALL GENKR
      CSKRX = COS(CKR)/CKR
      SNKRX = SIN(CKR)/CKR
      CKR2 = CKR*CKR
C     GENERATE BESSEL FUNCTIONS,THEIR DERIVATIVES AND NEUMANN FUNCTIONS.
      CALL GENBSL
C     PERFORM BESSEL TEST AND NEUMANN TEST
      QUANBT = ABS(CKR2*(BSSLSP(2)*CNEUMN(1)-BSSLSP(1)*CNEUMN(2))-1.0)
      QUANNT = ABS(CKR2*(BSSLSP(NRANKI)*CNEUMN(NRANK)-BSSLSP(NRANK)*CNEU
     1MN(NRANKI))-1.0)
      IF(QUANBT-1.0E-10)360,352,352
  352 THTPRT = RTD*THETA
      PRINT 356,THTPRT,CKR,QUANBT,QUANNT
  356 FORMAT(1H010X,13H***** THETA =F9.4,6H, KR =F10.4,15H, BESSEL TEST
     1=E12.5,16H, NEUMANN TEST =E12.5,6H *****)
      GO TO 362
  360 IF(QUANNT-1.0E-10)362,352,352
  362 CROW = 0.0
      CROWM = CMV
      IMR = 2
      DO 600 IROW = 1,NRANK
      CROW = CROW&1.0
      CROWM = CROWM&1.0
C     SET UP A LOOP FOR EACH COLUMN OF THE MATRICES.
      CCOL = 0.0
      CCOLM = CMV
      GO TO (364,366),IMR
  364 JMR = 1
      IMR = 2
      GO TO 368
  366 JMR = 2
      IMR = 1
  368 DO 400 ICOL = 1,NRANK
      CCOL = CCOL&1.0
      CRIJ = CROW&CCOL
      CRSSIJ = CROW*CCOL
      CCOLM = CCOLM&1.0
      CCOL1 = CCOL&1.0
      IF(IBODY-7)372,369,372
  369 IF(ICOL-IROW)370,372,372
  370 GO TO (390,398),JMR
  372 BJ1XP = CCOL1*DCKR*BSSLSP(ICOL&1)*SINTH
```

```
      BJBJ1 = CKR*(CKR*BSSLSP(ICOL)-CCOL*BSSLSP(ICOL&1))
      IF(IBODY-9)374,376,374
  374 GO TO (376,392),JMR
C     TEST FOR M = 0.
  376 IF(CMV)388,388,378
C     CALCULATE THE TERM FOR THE CURRENT ELEMENT IN THE I MATRIX.
  378 TERMI = SINTH*CKR2*BSSLSP(ICOL&1)*(COSTH*PNMLLG(IROW&1)*PNMLLG(ICO
     1L&1)*CRIJ-CROWM*PNMLLG(ICOL&1)*PNMLLG(IROW)-CCOLM*PNMLLG(IROW&1)*P
     2NMLLG(ICOL))
      AMXII(IROW,ICOL) = AMXII(IROW,ICOL)&SRMUL*CNFUMN(IROW&1)*TERMI
      IF(ICOL-IROW)388,384,384
  384 AMXIR(IROW,ICOL) = AMXIR(IROW,ICOL)&SRMUL*BSSLSP(IROW&1)*TERMI
  388 IF(IBODY-9)390,392,390
  390 JMR = 2
      GO TO 400
C     CALCULATE TERM FOR CURRENT ELEMENT IN THE J MATRIX.
  392 PTJ1 = PNMLLG(IROW&1)*PNMLLG(ICOL&1)*(BJBJ1*(CM2&CRSSIJ*COSTH**2)
     1&CRSSIJ*COSTH*BJ1XP)
      PTJ2 = CROWM*CCOL*PNMLLG(IROW)*PNMLLG(ICOL&1)*(COSTH*BJBJ1&BJ1XP)
      PTJ3 = CCOLM*PNMLLG(ICOL)*BJBJ1*(CROWM*PNMLLG(IROW)-CROW*COSTH*PNM
     1LLG(IROW&1))
      AMXJI(IROW,ICOL) = AMXJI(IROW,ICOL)&SRMUL*SINTH*CNFUMN(IROW&1)*(PT
     1J1-PTJ2&PTJ3)
      IF(ICOL-IROW)398,396,396
  396 AMXJR(IROW,ICOL) = AMXJR(IROW,ICOL)&SRMUL*SINTH*BSSLSP(IROW&1)*(PT
     1J1-PTJ2&PTJ3)
  398 JMR = 1
  400 CONTINUE
  600 CONTINUE
  700 CONTINUE
  800 CONTINUE
C     SYMMETRIZE REAL MATRICES AND IMAGINARY SPHERICAL MATRICES.
      DO 816 IROW = 2,NRANK
      IENDSY = IROW-1
      DO 812 ICOL = 1,IENDSY
      AMXIR(IROW,ICOL) = AMXIR(ICOL,IROW)
      AMXJR(IROW,ICOL) = AMXJR(ICOL,IROW)
C     TEST FOR SPHERICAL BODIES.
      IF(IBODY-7)812,808,812
  808 AMXII(IROW,ICOL) = AMXII(ICOL,IROW)
      AMXJI(IROW,ICOL) = AMXJI(ICOL,IROW)
  812 CONTINUE
  816 CONTINUE
C     SUMMATION FOR ALL MATRIX ELEMENTS COMPLETE. FINISH PROCESSING THEM
      DO 860 JROW = 1,NRANK
      DO 820 JCOL = 1,NRANK
      AMXIR(JROW,JCOL) = CMV*AMXIR(JROW,JCOL)
      AMXII(JROW,JCOL) = CMV*AMXII(JROW,JCOL)
      AMXJR(JROW,JCOL) = QEM*AMXJR(JROW,JCOL)
      AMXJI(JROW,JCOL) = QEM*AMXJI(JROW,JCOL)
C     COMPUTE K MATRIX AS A FUNCTION OF THE J MATRIX.
      AMXKR(JROW,JCOL) = -AMXJR(JROW,JCOL)
      AMXKI(JROW,JCOL) = -AMXJI(JROW,JCOL)
C     CALCULATE THE L MATRIX AS A FUNCTION OF THE I MATRIX.
      AMXLR(JROW,JCOL) = -AMXIR(JROW,JCOL)
      AMXLI(JROW,JCOL) = -AMXII(JROW,JCOL)
  820 CONTINUE
C     COMPUTE ADDITIONAL TERM FOR THE IMAGINARY PART OF THE K MATRIX.
      CKROW = JROW
      IF(KMV)824,824,826
```

```
  824 FCTKI = 1.0
      GO TO 840
  826 IF(JROW-KMV)828,830,830
  828 CMXNRM(JROW) = 1.0
      GO TO 860
  830 IBFCT = JROW-KMV&1
      IEFCT = JROW&KMV
      FPROD = IBFCT
      FCTKI = 1.0
      DO 832 LFCT = IBFCT,IEFCT
      FCTKI = FCTKI*FPROD
      FPROD = FPROD&1.0
  832 CONTINUE
  840 CMXNRM(JROW) = 4.0*CKROW*(CKROW&1.0)*FCTKI/(FM*(CKROW&CKROW&1.0))
      AMXKI(JROW,JROW) = AMXKI(JROW,JROW)&BDYFCT*CMXNRM(JROW)
      CMXNRM(JROW) = SQRT(CMXNRM(JROW))
  860 CONTINUE
C     PROCESS COMPUTED MATRICES
      CALL PRCSSM
  900 CONTINUE
      GO TO 20
      END
```

```
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C       A PROGRAM TO READ INPUT DATA FOR THE SCATTERING PROGRAM.
        SUBROUTINE RDDATA
        COMMON DTR,RTD,CPI
        COMMON /CMVCOM/ NM,CMI(30),CMV,KMV,CM2,EM,OFM,TWM,PRODM
        COMMON /FNCCOM/ PNMLLG(61),BSSLSP(61),CNFUMN(61)
        COMMON /MTXCOM/ NRANK,NRANKI,CMTXRI(120,120),CMTXIM(120,120),SPRMT
       1X(120,120),CMXNRM(60)
        COMMON /THTCOM/ THETA,NTHETA,DLTHTA,SINTH,COSTH,ISMRI,ISWTCH(7),SR
       1MUL,SMULSS(7),CDH(6),DHM,NSECT,NDPS(6),EPPS(6),KSECT
        COMMON /BDYCOM/ CKR,DCKR,CKR2,CSKRX,SNKRX,CONK,BRXT,ALPHA,IBODY,QB
       1,SNALPH,CSALPH
        COMMON /TOTCOM/ ACANS(60,10),STSFCT,RTSFCT
        COMMON /UVCCOM/ UANG(60),NUANG
        DIMENSION CLRTOT(600)
        EQUIVALENCE (ACANS(1,1),CLRTOT)
        DIMENSION EPDEG(10)
C       READ NECESSARY INPUT DATA
        PRINT 40
   40 FORMAT(1H1119X,3HCVM///////////////////////////1H039X,40H***********
      1*****************************/1H028X,63HELECTROMAGNETIC SCATTERING
      2 FROM GENERAL AXISYMMETRIC CONDUCTORS/1H039X,40H******************
      3**********************)
        READ 80,NM,NRANK,NSECT,IBODY,NUANG
   80 FORMAT(6I12)
        NRANKI = NRANK&1
        PRINT 88
   88 FORMAT(1H129X,75H              CASES     MATRIX RANK        SECTIONS
      1 BODY SHAPE         U VECTOR)
        PRINT 92,NM,NRANK,NSECT,IBODY,NUANG
   92 FORMAT(1H029X,5I15)
        READ 96,CONK,BRXT,ALPHA
   96 FORMAT(6F12.1)
        RTSFCT = 8.0/CONK
        STSFCT = 2.0*RTSFCT/CONK
        PRINT 100
  100 FORMAT(1H029X,60HBODY PARAMETERS               K(A)         BETA/RHO
     1        ALPHA)
        PRINT 104,CONK,BRXT,ALPHA
  104 FORMAT(1H044X,3F15.3)
        READ 96,(CMI(I),I = 1,NM)
        READ 80,(NDPS(I),I = 1,NSECT)
        PRINT 120,(NDPS(I),I = 1,NSECT)
  120 FORMAT(24H0    INTEGRATIONS/SECTION8I12,/(1H023X,8I12))
        READ 96,(UANG(I),I = 1,NUANG)
C       CLEAR AREA WHICH CONTAINS RUNNING TOTALS.
        DO 136 I = 1,600
        CLRTOT(I) = 0.0
  136 CONTINUE
C       COMPUTE END POINTS FOR THETA.
        ALPHA = DTR*ALPHA
        CALL CALENP
        DO 140 I = 1,NSECT
        EPDEG(I) = RTD*EPPS(I)
  140 CONTINUE
        PRINT 148,(EPDEG(I),I = 1,NSECT)
  148 FORMAT(24H0               END POINTS8F12.4,/(1H023X,8F12.4))
        RETURN
        END
```

```
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C     A ROUTINE TO COMPUTE A BESSEL FUNCTION OF SET ORDER AND ARGUMENT.
      SUBROUTINE BESSEL(NORDER,ARGMNT,ANSWR,IFRROR)
      DOUBLE PRECISION ARGMNT,ANSWR,X,CN,SUM,APR,TOPR,CI,CNI,ACR,PROD,
     1FACT
      IFRROR = 0
      N = NORDER
      X = ARGMNT
      CN = N
      SUM = 1.0
      APR = 1.0
      TOPR = -0.5D0*X*X
      CI = 1.0
      CNI = 2*N&3
      DO 60 I = 1,100
      ACR = TOPR*APR/(CI*CNI)
      SUM = SUM&ACR
      IF(DABS(ACR/SUM)-1.0D-20)100,100,40
   40 APR = ACR
      CI = CI&1.0D0
      CNI = CNI&2.0D0
   60 CONTINUE
      IFRROR = 1
      GO TO 200
C     THE SERIES HAS CONVERGED.
  100 PROD = 2*N&1
      FACT = 1.0
      IF(N)160,160,120
  120 DO 140 IFCT = 1,N
      FACT = FACT*X/PROD
      PROD = PROD-2.0D0
  140 CONTINUE
  160 ANSWR = FACT*SUM
  200 RETURN
      END
```

```
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C       A ROUTINE TO GENERATE LEGENDRE POLYNOMIALS.
        SUBROUTINE GENLGP
        COMMON OTR,RTD,CPI
        COMMON /CMVCOM/ NM,CMI(30),CMV,KMV,CM2,EM,QEM,TWM,PRODM
        COMMON /FNCCOM/ PNMLLG(61),BSSLSP(61),CNFUMN(61)
        COMMON /MTXCOM/ NRANK,NRANKI,CMTXRL(120,120),CMTXIM(120,120),SPRMT
       1X(120,120),CMXNRM(60)
        COMMON /THTCOM/ THETA,NTHETA,DLTHTA,SINTH,COSTH,ISMRL,ISWTCH(7),SR
       1MUL,SMULSS(7),CDH(6),DHM,NSECT,NDPS(6),EPPS(6),KSECT
        COMMON /BOYCOM/ CKR,DCKR,CKR2,CSKRX,SNKRX,CONK,BRXT,ALPHA,IBODY,OB
       1,SNALPH,CSALPH
        DTWM = TWM&1.0
        IF(THETA)16,4,16
      4 IF(KMV-1)6,12,6
      6 DO 8 ILG = 1,NRANKI
        PNMLLG(ILG) = 0.0
      8 CONTINUE
        GO TO 88
     12 PNMLLG(1) = 0.0
        PNMLLG(2) = 1.0
        PLA = 1.0
        GO TO 48
     16 IF(KMV)20,20,40
C       THE SPECIAL CASE WHEN M = 0.
     20 PLA = 1.0/SINTH
        PLB = COSTH*PLA
        PNMLLG(1) = PLA
        PNMLLG(2) = PLB
        IBEG = 3
        GO TO 60
C       GENERAL CASE FOR M NOT EQUAL TO 0.
     40 DO 44 ILG = 1,KMV
        PNMLLG(ILG) = 0.0
     44 CONTINUE
        PLA = PRODM*SINTH**(KMV-1)
        PNMLLG(KMV&1) = PLA
     48 PLB = DTWM*COSTH*PLA
        PNMLLG(KMV&2) = PLB
        IBEG = KMV&3
C       DO RECURSION FORMULA FOR ALL REMAINING LEGENDRE POLYNOMIALS.
     60 CNMUL = IBEG&IBEG-3
        CNM = 2.0
        CNMM = DTWM
        DO 80 ILGR = IBEG,NRANKI
        PLC = (CNMUL*COSTH*PLB-CNMM*PLA)/CNM
        PNMLLG(ILGR) = PLC
        PLA = PLB
        PLB = PLC
        CNMUL = CNMUL&2.0
        CNM = CNM&1.0
        CNMM = CNMM&1.0
     80 CONTINUE
     88 RETURN
        END
```

```
T         SUBTYPE,FORTRAN,LMAP,LSTRAP
C      A ROUTINE TO DO FINAL PROCESSING ON THE SCATTERING MATRIX.
       SUBROUTINE ADDPRC
       COMMON OTR,RTD,CPI
       COMMON /CMVCOM/ NM,CMI(30),CMV,KMV,CM2,EM,QEM,TWM,PRODM
       COMMON /FNCCOM/ PNMLLG(61),BSSLSP(61),CNEUMN(61)
       COMMON /MTXCOM/ NRANK,NRANKI,QMTXII(60,60),QMTXJI(60,60),QMTXKI(60
      1,60),QMTXLI(60,60),PMX1(60,60),PMX2(60,60),PMX3(60,60),PMX4(60,60)
      2,QMTXIR(60,60),QMTXJR(60,60),QMTXKR(60,60),QMTXLR(60,60),CMXNRM(60
      3)
       COMMON /VCMCOM/ ISYBG,JSYBG,KSYBG,NSYMT
       DIMENSION FGVECT(2,120,2),TCMPLX(2,120,120),FGMUL(120,2),FGANS(60,
      110)
       EQUIVALENCE (QMTXII,FGVECT),(PMX1,TCMPLX),(QMTXJI,FGMUL),(QMTXKI,F
      1GANS)
       COMMON /THTCOM/ THETA,NTHETA,DLTHTA,SINTH,COSTH,ISMRL,ISWTCH(7),SR
      1MUL,SMULSS(7),CDH(6),DHM,NSECT,NDPS(6),EPPS(6),KSECT
       COMMON /TOTCOM/ ACANS(60,10),STSECT,RTSECT
       COMMON /UVCCOM/ UANG(60),NUANG
       COMMON /BCYCOM/ CKR,DCKR,CKR2,CSKRX,SNKRX,CONK,BRXT,ALPHA,IBODY,QB
      1,SNALPH,CSALPH
C      NORMALIZE AND STORE SECTIONS T1 AND T3 OF THE COMPLEX T MATRIX.
       DO 40 IC = 1,NRANK
       DO 20 IR = 1,NRANK
       JR = IR&NRANK
       QUANNM = CMXNRM(IR)/CMXNRM(IC)
       TCMPLX(1,IR,IC) = QUANNM*QMTXIR(IR,IC)
       TCMPLX(1,JR,IC) = QUANNM*QMTXKR(IR,IC)
       TCMPLX(2,IR,IC) = QUANNM*QMTXII(IR,IC)
       TCMPLX(2,JR,IC) = QUANNM*QMTXKI(IR,IC)
   20  CONTINUE
   40  CONTINUE
C      NORMALIZE AND STORE SECTIONS T2 AND T4 OF THE COMPLEX T MATRIX.
       DO 80 IC = 1,NRANK
       JC = IC&NRANK
       DO 60 IR = 1,NRANK
       JR = IR&NRANK
       QUANNM = CMXNRM(IR)/CMXNRM(IC)
       TCMPLX(1,IR,JC) = QUANNM*QMTXJR(IR,IC)
       TCMPLX(1,JR,JC) = QUANNM*QMTXLR(IR,IC)
       TCMPLX(2,IR,JC) = QUANNM*QMTXJI(IR,IC)
       TCMPLX(2,JR,JC) = QUANNM*QMTXLI(IR,IC)
   60  CONTINUE
   80  CONTINUE
C      SET UP A LOOP FOR ALL VALUES OF THE ANGLE U.
       DO 400 IU = 1,NUANG
C      GENERATE LEGENDRE POLYNOMIALS AND DERIVITIVES. RESET THE LIST.
       IF(UANG(IU))96,88,96
   88  COSTH = 1.0
   92  SINTH = 0.0
       THETA = 0.0
       GO TO 112
   96  IF(UANG(IU)-180.0)104,100,104
  100  COSTH = -1.0
       GO TO 92
  104  THETA = OTR*UANG(IU)
       SINTH = SIN(THETA)
       COSTH = COS(THETA)
  112  CALL GENLGP
       DO 120 IPS = 1,NRANK
```

```
      FGMUL(IPS,1) = CMV*PNMLLG(IPS&1)
      CPS = IPS
      FGMUL(IPS,2) = CPS*COSTH*PNMLLG(IPS&1)-(CPS&CMV)*PNMLLG(IPS)
      JPS = IPS&NRANK
      FGMUL(JPS,1) = FGMUL(IPS,2)
      FGMUL(JPS,2) = FGMUL(IPS,1)
  120 CONTINUE
C     MULTIPLY THE T COMPLEX MATRIX TIMES THE LEGENDRE VECTORS.
      KMVM1 = (KMV-1)/4
      KMVM1 = 4*KMVM1
      IF(KMVM1)132,132,124
  124 DO 128 IZ = 1,KMVM1
      FGVECT(1,IZ,1) = 0.0
      FGVECT(2,IZ,1) = 0.0
      FGVECT(1,IZ,2) = 0.0
      FGVECT(2,IZ,2) = 0.0
      JZ = IZ&NRANK
      FGVECT(1,JZ,1) = 0.0
      FGVECT(2,JZ,1) = 0.0
      FGVECT(1,JZ,2) = 0.0
      FGVECT(2,JZ,2) = 0.0
  128 CONTINUE
      ISYBG = 2&2*(KMVM1)
      JSYBG = KMVM1
      KSYBG = 2*KMVM1
      NSYMT = NRANK-KMVM1
      GO TO 136
  132 ISYBG = 0
      JSYBG = 0
      KSYBG = 0
      NSYMT = NRANK
  136 CALL VECMUL
C     A LOOP TO ZERO CURRENT SUMS OF SCAT1,2, TOTAL1,2 AND RTRAD1,2.
      DO 140 IZ = 1,10
      FGANS(IU,IZ) = 0.0
  140 CONTINUE
C     SET UP LOOP FOR CURRENT VALUES OF THE SUMS.
      IPTH = 1
      DO 200 ICMS = 1,NRANK
      JCMS = ICMS&NRANK
C     COMPUTE SCATT1 AND SCATT2 SUMS
      FGANS(IU,1) = FGANS(IU,1)&(FGVECT(1,ICMS,1)**2&FGVECT(2,ICMS,1)**2
     1&FGVECT(1,JCMS,1)**2&FGVECT(2,JCMS,1)**2)/CMXNRM(ICMS)**2
      FGANS(IU,6) = FGANS(IU,6)&(FGVECT(1,JCMS,2)**2&FGVECT(2,JCMS,2)**2
     1&FGVECT(1,ICMS,2)**2&FGVECT(2,ICMS,2)**2)/CMXNRM(ICMS)**2
C     FORM THE REAL AND IMAGINARY PARTS OF TOTAL1,2 AND RTRAD 1,2
      PFR1 = FGVECT(1,ICMS,1)*FGMUL(ICMS,1)
      PFI1 = FGVECT(2,ICMS,1)*FGMUL(ICMS,1)
      PFR2 = -FGVECT(2,JCMS,2)*FGMUL(ICMS,1)
      PFI2 = FGVECT(1,JCMS,2)*FGMUL(ICMS,1)
      PGR1 = FGVECT(1,JCMS,1)*FGMUL(JCMS,1)
      PGI1 = FGVECT(2,JCMS,1)*FGMUL(JCMS,1)
      PGR2 = -FGVECT(2,ICMS,2)*FGMUL(JCMS,1)
      PGI2 = FGVECT(1,ICMS,2)*FGMUL(JCMS,1)
      GO TO (150,154,158,162),IPTH
  150 SGN = &1.0
      IPTH = 2
      GO TO 170
  154 SGN = -1.0
      IPTH = 3
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```
      GO TO 180
  158 SGN = -1.0
      IPTH = 4
      GO TO 170
  162 SGN = &1.0
      IPTH = 1
      GO TO 180
C     CASE FOR N MOD 4 IS 1 (-I,&I) OR 3 (&I,-I)
  170 FGANS(IU,2) = FGANS(IU,2)&SGN*(PFI1&PGR1)/CMXNRM(ICMS)**2
      FGANS(IU,3) = FGANS(IU,3)-SGN*(PFR1-PGI1)/CMXNRM(ICMS)**2
      FGANS(IU,7) = FGANS(IU,7)&SGN*(PFI2-PGR2)/CMXNRM(ICMS)**2
      FGANS(IU,8) = FGANS(IU,8)-SGN*(PFR2&PGI2)/CMXNRM(ICMS)**2
      FGANS(IU,4) = FGANS(IU,4)-SGN*(PFI1-PGR1)/CMXNRM(ICMS)**2
      FGANS(IU,5) = FGANS(IU,5)&SGN*(PFR1&PGI1)/CMXNRM(ICMS)**2
      FGANS(IU,9) = FGANS(IU,9)-SGN*(PFI2&PGR2)/CMXNRM(ICMS)**2
      FGANS(IU,10) = FGANS(IU,10)&SGN*(PFR2-PGI2)/CMXNRM(ICMS)**2
      GO TO 200
C     CASE FOR N MOD 4 IS 2 (-1,-1) OR 4 (&1,&1)
  180 FGANS(IU,2) = FGANS(IU,2)&SGN*(PFR1-PGI1)/CMXNRM(ICMS)**2
      FGANS(IU,3) = FGANS(IU,3)&SGN*(PFI1&PGR1)/CMXNRM(ICMS)**2
      FGANS(IU,7) = FGANS(IU,7)&SGN*(PFR2&PGI2)/CMXNRM(ICMS)**2
      FGANS(IU,8) = FGANS(IU,8)&SGN*(PFI2-PGR2)/CMXNRM(ICMS)**2
      FGANS(IU,4) = FGANS(IU,4)&SGN*(PFR1&PGI1)/CMXNRM(ICMS)**2
      FGANS(IU,5) = FGANS(IU,5)&SGN*(PFI1-PGR1)/CMXNRM(ICMS)**2
      FGANS(IU,9) = FGANS(IU,9)&SGN*(PFR2-PGI2)/CMXNRM(ICMS)**2
      FGANS(IU,10) = FGANS(IU,10)&SGN*(PFI2&PGR2)/CMXNRM(ICMS)**2
  200 CONTINUE
C     SCALE ACCUMULATIVE SUMS
      FGANS(IU,1) = STSFCT*FGANS(IU,1)
      FGANS(IU,2) = STSFCT*FGANS(IU,2)
      FGANS(IU,3) = STSFCT*FGANS(IU,3)
      FGANS(IU,4) = RTSFCT*FGANS(IU,4)
      FGANS(IU,5) = RTSFCT*FGANS(IU,5)
      FGANS(IU,6) = STSFCT*FGANS(IU,6)
      FGANS(IU,7) = STSFCT*FGANS(IU,7)
      FGANS(IU,8) = STSFCT*FGANS(IU,8)
      FGANS(IU,9) = RTSFCT*FGANS(IU,9)
      FGANS(IU,10) = RTSFCT*FGANS(IU,10)
  400 CONTINUE
C     PRINT PARTIAL SUMS AND ACCUMULATE TOTALS.
      DO 500 IPR = 1,2
      PRINT 420,KMV,IPR
  420 FORMAT(1H137X,31H********** CURRENT SUMS FOR M =I3,11H **********/
     1/6HOCLASSI2,82H  ANGLE      SCATTERING     TOTAL(REAL)     TOTAL(IMAG
     2)     RTRAD(REAL)     RTRAD(IMAG)//)
      IBEG = 1&5*(IPR-1)
      IEND = IBEG&4
      DO 460 IUP = 1,NUANG
      DO 432 ICAL = IBEG,IEND
      ACANS(IUP,ICAL) = ACANS(IUP,ICAL)&FGANS(IUP,ICAL)
  432 CONTINUE
      PRINT 440,UANG(IUP),(FGANS(IUP,LP),LP = IBEG,IEND)
  440 FORMAT(1H F14.2,1P7E15.6)
  460 CONTINUE
  500 CONTINUE
C     PRINT THE ACCUMULATE TOTALS.
      DO 600 JPR = 1,2
      PRINT 520,KMV,JPR
  520 FORMAT(1H135X,35H*********** ACCUMULATED SUMS FOR M =I3,11H *******
     1***//6HOCLASSI2,112H  ANGLE      SCATTERING     TOTAL(REAL)     TOTAL
```

```
     2(IMAG)     RTRAD(REAL)     RTRAD(IMAG)              RCS     PHASE ANGLE
     3//)
      JBEG = 1&5*(JPR-1)
      JEND = JBEG&4
      DO 560 JUP = 1,NUANG
C     COMPUTE PHASE ANGLE AND RCS 1 OR 2.
      PHANG = RTD*ATAN2(ACANS(JUP,JEND),ACANS(JUP,JEND-1))
      RCS12 = ACANS(JUP,JEND-1)**2&ACANS(JUP,JEND)**2
      PRINT 440,UANG(JUP),(ACANS(JUP,LP),LP = JBEG,JEND),RCS12,PHANG
  540 FORMAT(1H F14.3,1P7E15.6)
  560 CONTINUE
  600 CONTINUE
      RETURN
      END
```

```
T         SUBTYPE,FORTRAN,LMAP,LSTRAP
C      A SUBROUTINE TO PRINT OUT THE T MATRIX.
       SUBROUTINE PRTRIT
       COMMON /MTXCOM/ NRANK,NRANKI,QMTXII(60,60),QMTXJI(60,60),QMTXKI(60
      1,60),QMTXLI(60,60),PMX1(60,60),PMX2(60,60),PMX3(60,60),PMX4(60,60)
      2,QMTXIR(60,60),QMTXJR(60,60),QMTXKR(60,60),QMTXLR(60,60),CMXNRM(60
      3)
       EQUIVALENCE (QMTXII,CMTXRL),(PMX1,CMTXIM),(QMTXIR,SPRMTX)
       DIMENSION CMTXRL(120,120),CMTXIM(120,120),SPRMTX(120,120)
       PRINT 28
    28 FORMAT(1H1///1H052X,16HMATRIX T(1),REAL)
       CALL PRINTM(QMTXIR,NRANK)
       PRINT 128
   128 FORMAT(1H1///1H052X,16HMATRIX T(2),REAL)
       CALL PRINTM(QMTXJR,NRANK)
       PRINT 228
   228 FORMAT(1H1///1H052X,16HMATRIX T(3),REAL)
       CALL PRINTM(QMTXKR,NRANK)
       PRINT 328
   328 FORMAT(1H1///1H052X,16HMATRIX T(4),REAL)
       CALL PRINTM(QMTXLR,NRANK)
       PRINT 428
   428 FORMAT(1H1///1H049X,21HMATRIX T(1),IMAGINARY)
       CALL PRINTM(QMTXII,NRANK)
       PRINT 528
   528 FORMAT(1H1///1H049X,21HMATRIX T(2),IMAGINARY)
       CALL PRINTM(QMTXJI,NRANK)
       PRINT 628
   628 FORMAT(1H1///1H049X,21HMATRIX T(3),IMAGINARY)
       CALL PRINTM(QMTXKI,NRANK)
       PRINT 728
   728 FORMAT(1H1///1H049X,21HMATRIX T(4),IMAGINARY)
       CALL PRINTM(QMTXLI,NRANK)
       RETURN
       END
```

```
T          SUBTYPE,STRAP
 *         A ROUTINE TO GET THE F1,G1 AND F2,G2 VECTORS FROM T * PICOS (U).
           PUNREL
           PUNFPC,LAST,COMMUN
 VECMUL    ENTER,SVXRS
           PUNCDC
 MTXCOM    COMBLOCK,FINAL
 VCMCOM    COMBLOCK,FINALV
           XW,0,0,0
 SVXRS     SX,$12,XR12               *SAVE INDEX REGISTERS 12,
           SX,$13,XR13               *                             13,
           SX,$14,XR14               *                                14,
           SX,$2,XR2
           SX,$3,XR3
           SX,$10,XR10
           LWF(U),TWERTY             *COMPUTE ADDRESS FACTOR FOR G1 OR F2.
           D*(U),NRANK
           SHFL,38
           F-I(U),38
           ST(U),G1F2A
           LX,$12,TMXWR              *RESET T COMPLEX MATRIX CONTROL.
           V&,$12,ISYBG&0.32
           LCI,$12,2.0
           SX,$12,TMXW
           LX,$13,PMXWR              *RESET MULTIPLIER VECTOR CONTROL.
           V&,$13,JSYBG&0.32
           LC,$13,NSYMT&0.32
           SX,$13,PMXW
           L(U),NSYMT
           &(U),NSYMT
           LX,$14,FGXWR
           V&,$14,KSYBG&0.32
           LC,$14,$L&0.32
           SX,$14,FGXW
           FZ(BU,1,8),REGGF&0.20     *SET SIGN FOR ANSWER STORAGE, F1 OR G1
           FZ(BU,1,8),IEGGF&0.20
 *         DO THE REAL PART OF F1 OR G1, G2 OR F2.
 BGANLP    LX,$2,$12
           V&,$2,G1F2A&0.32
           LX,$3,$13
           V&,$3,NRANK&0.32
 SRLF1R    KC,$14,NSYMT&0.32
           BZXE,NOCMPR
           V&,$12,KSYBG&0.32
           SV,$12,TMXW
           V&,$14,KSYBG&0.32
           LV,$2,$12
           V&,$2,G1F2A&0.32
 NOCMPR    LVI,$10,N1F1R
           DL(U),ZERO
 SMLF1R    B,0.0($10)
 N1F1R     LMR(U),0.0($12)           *N MOD 4 = I OR 1.0
           *&(N),0.0($13)
           LMR(U),1.0($2)
           *&(N),0.0($3)
           LVI,$10,N2F1R
           B,FMLF1R
 N2F1R     LMR(U),1.0($12)           *N MOD 4 = -1 OR I
           *N&(N),0.0($13)
           LMR(U),0.0($2)
```

```
         *&(N),0.0($3)
         LVI,$10,N3F1R
         B,EMLF1R
N3F1R    LMR(U),0.0($12)          'N MOD 4 = -I OR -1.0
         *N&(N),0.0($13)
         LMR(U),1.0($2)
         *N&(N),0.0($3)
         LVI,$10,N4F1R
         B,EMLF1R
N4F1R    LMR(U),1.0($12)          'N MOD 4 = 1 OR -I
         *&(N),0.0($13)
         LMR(U),0.0($2)
         *N&(N),0.0($3)
         LVI,$10,N1F1R
EMLF1R   V&I,$12,MTXSZE
         V&I,$2,MTXSZE
         V&I,$13,1.0
         V&I,$3,1.0
         CBR,$13,SMLF1R
REGGF    SRD(N),0.0($14)          'STORE REAL PART OF F1 OR G1, G2 OR F2
         V&I,$14,1.0
*        DO THE IMAGINARY PART OF F1 OR G1, G2 OR F2.
         LV,$3,$13
         V&,$3,NRANK&0.32
         LV,$12,TMXW
         LV,$2,$12
         V&,$2,G1F2A&0.32
         LVI,$10,N1F1I
         DL(U),ZERO
SMLF1I   B,0.0($10)               'N MOD 4 = I OR 1.0
N1F1I    LMR(U),1.0($12)
         *&(N),0.0($13)
         LMR(U),0.0($2)
         *N&(N),0.0($3)
         LVI,$10,N2F1I
         B,EMLF1I
N2F1I    LMR(U),0.0($12)          'N MOD 4 = -1.0 OR I
         *&(N),0.0($13)
         LMR(U),1.0($2)
         *&(N),0.0($3)
         LVI,$10,N3F1I
         B,EMLF1I
N3F1I    LMR(U),1.0($12)          'NMOD 4 = -I OR -1.0
         *N&(N),0.0($13)
         LMR(U),0.0($2)
         *&(N),0.0($3)
         LVI,$10,N4F1I
         B,EMLF1I
N4F1I    LMR(U),0.0($12)           'N MOD 4 = 1.0 OR -I
         *N&(N),0.0($13)
         LMR(U),1.0($2)
         *N&(N),0.0($3)
         LVI,$10,N1F1I
EMLF1I   V&I,$12,MTXSZE
         V&I,$2,MTXSZE
         V&I,$13,1.0
         V&I,$3,1.0
         CBR,$13,SMLF1I
IFGGF    SRD(N),0.0($14)          'STORE IMAGINARY PART OF F1,G1,G2,F2.
         LV,$3,$13                'SET UP FOR NEXT ITEM.
```

```
         V&,$3,NRANK&0.32
         LV,$12,TMXW               *PREPARE FOR NEXT ROW OF T MATRIX.
         V&I,$12,2.0
         SV,$12,TMXW
         LV,$2,$12
         V&,$2,G1F2A&0.32
         CBR&,$14,SRLF1R
         V&I,$14,MTXSZF            *RESET XR14 FOR G2,F2 VECTOR
         LV,$12,TMXWR              *RESET TO FIRST ROW IN T MATRIX.
         V&,$12,ISYBG&0.32
         SV,$12,TMXW
         V&I,$13,VCTSZE            *RESET MULTIPLIER VECTOR
         SV,$13,PMXW
         F1(BU,1,8),REGGF&0.20     *SET SIGN FOR ANSWER STORAGE, F2 OR G2
         F1(BU,1,8),IFGGF&0.20
         CB,$12,BGANLP
         LX,$2,XR2
         LX,$3,XR3
         LX,$10,XR10
         LX,$12,XR12
         LX,$13,XR13
         LX,$14,XR14
         B,0.0($15)
*        STORAGE REQUIREMENTS FOR THE VECTOR MULTIPLICATION ROUTINE
XR12     XW,0,0,0                  *SAVE SPACE FOR INDEX REGISTER.
XR13     XW,0,0,0
XR14     XW,0,0,0
XR2      XW,0,0,0
XR3      XW,0,0,0
XR10     XW,0,0,0
TMXW     XW,TCMPLX,0,TMXW          *INDEX CONTROL FOR T COMPLEX MATRIX.
PMXW     XW,FGMUL,0,PMXW           *INDEX CONTROL FOR POLYNOMIAL VECTOR.
FGXW     XW,FGVECT,0,FGXW          *INDEX CONTROL FOR VECTOR ANSWERS.
TMXWR    XW,TCMPLX,0,TMXW
PMXWR    XW,FGMUL,0,PMXW
FGXWR    XW,FGVECT,0,FGXW
G1F2A    DRZ(U),1                  *STORAGE FOR INDEX INCREMENTER G1,F2.
TWFRTY   (F10)DD(U),240.0X38       *INTEGER = 240
ZERO     DD(N),0
MTXSZE   SYN,240.0
VCTSZE   SYN,120.0
LAST     SYN,$
         SLCRCOM
COMMON   SYN,$
         SLCRCOM,MTXCOM
NRANK    DR(U),1
NRANKI   CR(U),1
FGVECT   DR(N),480
QMIIRM   DR(N),3120
FGMUL    DR(N),240
QMJIRM   DR(N),3360
FGANS    DR(N),600
QMKIRM   DR(N),3000
QMLITL   CR(N),3600
TCMPLX   DR(N),28800
CMXNRM   DR(N),60
FINAL    SYN,$
         SLCRCOM,VCMCOM
ISYBG    DR(U),1
JSYBG    DR(U),1
```

```
KSYBG    OR(U),1
NSYMT    OR(U),1
FINALV   SYN,$
         END
```

```
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C       A ROUTINE TO PRINT OUT A MATRIX ARRAY
        SUBROUTINE PRINTM(P,N)
        DIMENSION P(60,60)
        NR = N
        DO 100 I = 1,NR
        IB = 1
   20   IE = IB&7
        IF(IE-NR)28,28,24
   24   IE = NR
   28   IF(IB-1)36,36,60
   36   PRINT 44,I,(P(I,J),J = IB,IE)
   44   FORMAT(5H0 ROWI3,2X,1P8F15.6)
        GO TO 80
   60   PRINT 68,(P(I,J),J = IB,IE)
   68   FORMAT(1H 9X,1P8F15.6)
   80   IB = IE&1
        IF(IB-NR)20,20,100
  100   CONTINUE
        RETURN
        END
        END
```

```
T         SUBTYPE,FORTRAN,LMAP,LSTRAP
C       A ROUTINE FOR BESSEL FUNCTIONS, DERIVATIVES AND NEUMANN FUNCTIONS.
        SUBROUTINE GENBSL
        COMMON DTR,RTD,CPI
        COMMON /CMVCOM/ NM,CMI(30),CMV,KMV,CM2,EM,OEM,TWM,PRODM
        COMMON /FNCCOM/ PNMLLG(61),BSSLSP(61),CNEUMN(61)
        COMMON /MTXCOM/ NRANK,NRANKI,CMTXRL(120,120),CMTXIM(120,120),SPRMT
       1X(120,120),CMXNRM(60)
        COMMON /THTCOM/ THETA,NTHETA,DLTHTA,SINTH,COSTH,ISMRL,ISWTCH(7),SR
       1MUL,SMULSS(7),CDH(6),DHM,NSECT,NDPS(6),EPPS(6),KSECT
        COMMON /BDYCOM/ CKR,OCKR,CKR2,CSKRX,SNKRX,CONK,BRXT,ALPHA,IBODY,OB
       1,SNALPH,CSALPH
        DOUBLE PRECISION PCKR,ANSWR,ANSA,ANSB,ANSC,CONN
C       SET UP A LOOP TO GET 2 SUCCESSIVE BESSEL FUNCTIONS.
        NVAL = NRANK-1
        PCKR = CKR
        DO 40 I = 1,4
        CALL BESSEL(NVAL,PCKR,ANSWR,IERROR)
        IF(IERROR)20,20,32
   20   ANSA = ANSWR
        NVAL = NVAL&1
        CALL BESSEL(NVAL,PCKR,ANSWR,IERROR)
        IF(IERROR)24,24,28
   24   ANSB = ANSWR
        GO TO 60
   28   NVAL = NVAL-1
   32   NVAL = NVAL&NRANK
   40   CONTINUE
C       PROGRAM UNABLE TO GENERATE BESSEL FUNCTION.
        CALL DUMP
C       SET UP FOR PROPER RECURSION OF THE BESSEL FUNCTIONS.
   60   IF(NVAL-NRANK)100,100,64
   64   IEND = NVAL-NRANK
        CONN = 2*(NVAL-1)&1
        DO 72 IP = 1,IEND
        ANSC = CONN*ANSA/PCKR-ANSB
        CONN = CONN-2.0D0
        ANSB = ANSA
        ANSA = ANSC
   72   CONTINUE
C       PROGRAM IS READY TO RECURSE DOWNWARD INTO BESSEL FUNCTION VECTOR.
  100   BSSLSP(NRANKI) = ANSB
        BSSLSP(NRANKI-1) = ANSA
        CONN = NRANK&NRANK-1
        IE = NRANKI-2
        JE = IE
        DO 120 JB = 1,JE
        ANSC = CONN*ANSA/PCKR-ANSB
        BSSLSP(IE) = ANSC
        ANSB = ANSA
        ANSA = ANSC
        IE = IE-1
        CONN = CONN-2.0D0
  120   CONTINUE
C       GENERATE THE NEUMANN FUNCTIONS.
        CMULN = 3.0
        SNSA = -CSKRX
        SNSB = -CSKRX/CKR-SNKRX
        CNEUMN(1) = SNSA
        CNEUMN(2) = SNSB
```

```
      DO 280 I = 3,NRANKI
      SNSC = CMULN*SNSB/CKR-SNSA
      CNFUMN(I) = SNSC
      SNSA = SNSB
      SNSB = SNSC
      CMULN = CMULN+2.0
  280 CONTINUE
      RETURN
      END
```

```
T          SUBTYPE,STRAP
 *         A ROUTINE TO DUMP CORE.
           PUNREL
           PUNEPC,LAST,COMMON
 DUMP      ENTER,START
           XW,0,0,0
 START     B,$MCP
           ,$ABEOJ
           B,0.0($15)                *RETURN
 LAST      SYN,$
           SLCRCOM,
 COMMON    SYN,$
           END
```

```
T         SUBTYPE,FORTRAN,LMAP,LSTRAP
C      A ROUTINE TO ORTHOGANILIZE THE Q MATRICES TO PRODUCE T MATRICES.
       SUBROUTINE PRCSSM
       COMMON /MTXCOM/ NR,NRI,QI1(60,60),QI2(60,60),QI3(60,60),QI4(60,60)
      1,P1(60,60),P2(60,60),P3(60,60),P4(60,60),QR1(60,60),QR2(60,60),QR3
      2(60,60),QR4(60,60),CMXNRM(60)
       EQUIVALENCE (QI1,RI1),(QR1,RR1),(TMMX,P1)
       DIMENSION RR1(120,120),RI1(120,120),TMMX(120,120)
C      NORMALIZE AND TRANSPOSE THE I,J,K,L MATRICES TO OBTAIN Q MATRICES.
       CALL NRMQMX
C      SET UP REAL AND IMAGINARY MATRICES FOR GENERAL M CASE.
       DO 6 I=1,NR
       MM=I&I
       DO 4 J=1,NR
       NN=J&J
       TMMX(MM-1,NN-1) = QI1(I,J)
       TMMX(MM-1,NN)   = QI2(I,J)
       TMMX(MM,NN-1)   = QI3(I,J)
       TMMX(MM,NN)     = QI4(I,J)
    4  CONTINUE
    6  CONTINUE
       NBGR = NR&NR
       DO 10 I = 1,NBGR
       DO 8 J = 1,NBGR
       RI1(I,J) = TMMX(I,J)
    8  CONTINUE
   10  CONTINUE
       DO 14 I = 1,NR
       MM = I&I
       DO 12 J = 1,NR
       NN = J&J
       TMMX(MM-1,NN-1) = QR1(I,J)
       TMMX(MM-1,NN)   = QR2(I,J)
       TMMX(MM,NN-1)   = QR3(I,J)
       TMMX(MM,NN)     = QR4(I,J)
   12  CONTINUE
   14  CONTINUE
       DO 18 I = 1,NBGR
       DO 16 J = 1,NBGR
       RR1(I,J) = TMMX(I,J)
   16  CONTINUE
   18  CONTINUE
C      CONDITION Q MATRICES BEFORE ORTHOGONALIZING THEM.
       CALL CNDTNQ
C      NORMALIZE THE NTH ROW OF AN N BY N MATRIX
       SUM1 = 0.0
       DO 20 K = 1,NBGR
       SUM1 = RR1(NBGR,K)**2&RI1(NBGR,K)**2&SUM1
   20  CONTINUE
       SUM1 = SQRT(SUM1)
       DO 28 K = 1,NBGR
       RR1(NBGR,K) = RR1(NBGR,K)/SUM1
       RI1(NBGR,K) = RI1(NBGR,K)/SUM1
   28  CONTINUE
C      SET UP A LOOP FOR THE N-1 REMAINING ROWS.
       NMI = NBGR-1
       NROW = NBGR
       DO 100 I = 1,NMI
       NROW = NROW-1
       MROW = NROW
```

```
      DO 36 K=1,NBGR
      TMMX(1,K) = RR1(NROW,K)
      TMMX(2,K) = RI1(NROW,K)
   36 CONTINUE
      DO 80 J = NROW,NMI
      SR1 = 0.0
      SI1 = 0.0
      MROW = MROW&1
      DO 40 K = 1,NBGR
      SR1 = SR1&RR1(MROW,K)*RR1(NROW,K)&RI1(MROW,K)*RI1(NROW,K)
      SI1 = SI1&RR1(MROW,K)*RI1(NROW,K)-RI1(MROW,K)*RR1(NROW,K)
   40 CONTINUE
      DO 48 K = 1,NBGR
      TMMX(1,K) = TMMX(1,K)-SR1*RR1(MROW,K)&SI1*RI1(MROW,K)
      TMMX(2,K) = TMMX(2,K)-SR1*RI1(MROW,K)-SI1*RR1(MROW,K)
   48 CONTINUE
   80 CONTINUE
      SUM1 = 0.0
      DO 84 K = 1,NBGR
      SUM1 = SUM1&TMMX(1,K)**2&TMMX(2,K)**2
   84 CONTINUE
      SUM1 = SQRT(SUM1)
      DO 88 K = 1,NBGR
      RR1(NROW,K) = TMMX(1,K)/SUM1
      RI1(NROW,K) = TMMX(2,K)/SUM1
   88 CONTINUE
  100 CONTINUE
C     PRINT OUT ORTHOGANILIZED Q MATRICES
      PRINT 120
  120 FORMAT (1H140X,40HREAL SECTION OF ORTHOGANILIZED Q MATRIX.)
      CALL PRNOOT(RR1,NBGR)
      PRINT 128
  128 FORMAT(1H137X,45HIMAGINARY SECTION OF ORTHOGANILIZED Q MATRIX.)
      CALL PRNOOT(RI1,NBGR)
C     PERFORM Q TRANSPOSE * REAL(Q) TO GET T MATRIX.
      DO 160 I = 1,NBGR
      DO 152 J = 1,NBGR
      TMMX(I,J) = 0.0
  152 CONTINUE
  160 CONTINUE
      DO 180 I = 1,NBGR
      DO 176 J = 1,NBGR
      DO 172 K = 1,NBGR
      TMMX(I,J) = TMMX(I,J)-RI1(K,I)*RR1(K,J)
  172 CONTINUE
  176 CONTINUE
  180 CONTINUE
      DO 196 I = 1,NR
      MM = I&I
      DO 192 J = 1,NR
      NN = J&J
      QI1(I,J) = TMMX(MM-1,NN-1)
      QI2(I,J) = TMMX(MM-1,NN)
      QI3(I,J) = TMMX(MM,NN-1)
      QI4(I,J) = TMMX(MM,NN)
  192 CONTINUE
  196 CONTINUE
      DO 208 I = 1,NBGR
      DO 204 J = 1,NBGR
      TMMX(I,J) = 0.0
```

```
  204 CONTINUE
  208 CONTINUE
      DO 220 I = 1,NBGR
      DO 216 J = 1,NBGR
      DO 212 K = 1,NBGR
      TMMX(I,J) = TMMX(I,J)&RRI(K,I)*RR1(K,J)
  212 CONTINUE
  216 CONTINUE
  220 CONTINUE
      DO 236 I = 1,NR
      MM = I&I
      DO 232 J = 1,NR
      NN = J&J
      QR1(I,J) = TMMX(MM-1,NN-1)
      QR2(I,J) = TMMX(MM-1,NN)
      QR3(I,J) = TMMX(MM,NN-1)
      QR4(I,J) = TMMX(MM,NN)
  232 CONTINUE
  236 CONTINUE
C     PRINT THE T MATRIX
      CALL PRTRIT
C     DO FINAL PROCESSING
      CALL ADDPRC
      RETURN
      END
```

```
T          SUBTYPE,FORTRAN,LMAP,LSTRAP
C      A ROUTINE TO CONDITION Q MATRICES BEFORE ORTHOGONALIZING THEM.
       SUBROUTINE CNDTNQ
       COMMON /MTXCOM/ NR,NRI,QI1(60,60),QI2(60,60),QI3(60,60),QI4(60,60)
      1,P1(60,60),P2(60,60),P3(60,60),P4(60,60),QR1(60,60),QR2(60,60),QR3
      2(60,60),QR4(60,60),CMXNRM(60)
       EQUIVALENCE (QI1,RI1),(QR1,RR1),(TMMX,P1)
       DIMENSION RR1(120,120), RI1(120,120),TMMX(120,120)
C      SET UP LOOPS FOR ALL BUT THE FIRST ROW.
       NBGR = NR&NR
       NROW = NBGR
       DO 60 KR = 2,NBGR
C      RESCALE THE CURRENT ROW.
       SCLE1 = 1.0/RI1(NROW,NROW)
       DO 8 LC = 1,NBGR
       RR1(NROW,LC) = SCLE1*RR1(NROW,LC)
       RI1(NROW,LC) = SCLE1*RI1(NROW,LC)
     8 CONTINUE
C      RESCALE ALL THE ROWS UP TO THE CURRENT ROW.
       MROW = NROW-1
       DO 20 MR = 1,MROW
       RSCL1 = RI1(MR,NROW)
       DO 16 MC = 1,NBGR
       RR1(MR,MC) = RR1(MR,MC)-RSCL1*RR1(NROW,MC)
       RI1(MR,MC) = RI1(MR,MC)-RSCL1*RI1(NROW,MC)
    16 CONTINUE
    20 CONTINUE
       NROW = NROW-1
    60 CONTINUE
C      SET IMAGINARY ELEMENTS ABOVE THE MAIN DIAGONAL = 0.
       NROW = NBGR-1
       DO 80 I = 1,NROW
       IB = I&1
       DO 72 J = IB,NBGR
       RI1(I,J) = 0.0
    72 CONTINUE
    80 CONTINUE
       RETURN
       END
```

```
T         SUBTYPE,FORTRAN,LMAP,LSTRAP
C      A ROUTINE TO NORMALIZE THE I,J,K AND L MATRICES TO GET A Q MATRIX.
       SUBROUTINE NRMQMX
       COMMON /MTXCOM/ NRANK,NRANKL,AMXIR(60,60),AMXJR(60,60),AMXKR(60,60
      1),AMXLR(60,60),AMXII(60,60),AMXJI(60,60),AMXKI(60,60),AMXLI(60,60)
      2,QMTXIR(60,60),QMTXJR(60,60),QMTXKR(60,60),QMTXLR(60,60),CMXNRM(60
      3)
       EQUIVALENCE (AMXIR,QMTXII),(AMXJR,QMTXJI),(AMXKR,QMTXKI),(AMXLR,QM
      1TXLI)
       DIMENSION QMTXII(60,60),QMTXJI(60,60),QMTXKI(60,60),QMTXLI(60,60)
C      SET UP LOOPS TO PROCESS ALL ROWS AND COLUMNS FOR THE REAL MATRICES
       DO 200 IR = 1,NRANK
       DO 100 IC = 1,NRANK
       QUANNM = CMXNRM(IR)*CMXNRM(IC)
       QMTXIR(IR,IC) = AMXJR(IC,IR)/QUANNM
       QMTXJR(IR,IC) = -AMXLR(IC,IR)/QUANNM
       QMTXKR(IR,IC) = AMXIR(IC,IR)/QUANNM
       QMTXLR(IR,IC) = AMXKR(IC,IR)/QUANNM
  100  CONTINUE
  200  CONTINUE
C      SET UP LOOPS OF ROWS AND COLUMNS FOR THE IMAGINARY MATRICES.
       DO 400 IR = 1,NRANK
       DO 300 IC = 1,NRANK
       QUANNM = CMXNRM(IR)*CMXNRM(IC)
       QMTXII(IR,IC) = AMXJI(IC,IR)/QUANNM
       QMTXJI(IR,IC) = -AMXLI(IC,IR)/QUANNM
       QMTXKI(IR,IC) = AMXII(IC,IR)/QUANNM
       QMTXLI(IR,IC) = AMXKI(IC,IR)/QUANNM
  300  CONTINUE
  400  CONTINUE
C      PRINT OUT NORMALIZED AND TRANSPOSED Q MATRICES
       PRINT 420
  420  FORMAT(1H140X,38HREAL PART OF Q1(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXIR,NRANK)
       PRINT 428
  428  FORMAT(1H140X,38HREAL PART OF Q2(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXJR,NRANK)
       PRINT 436
  436  FORMAT(1H140X,38HREAL PART OF Q3(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXKR,NRANK)
       PRINT 444
  444  FORMAT(1H140X,38HREAL PART OF Q4(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXLR,NRANK)
       PRINT 452
  452  FORMAT(1H138X,43HIMAGINARY PART OF Q1(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXII,NRANK)
       PRINT 460
  460  FORMAT(1H138X,43HIMAGINARY PART OF Q2(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXJI,NRANK)
       PRINT 468
  468  FORMAT(1H138X,43HIMAGINARY PART OF Q3(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXKI,NRANK)
       PRINT 476
  476  FORMAT(1H138X,43HIMAGINARY PART OF Q4(NORMALIZED,TRANSPOSED))
       CALL PRINTM(QMTXLI,NRANK)
       RETURN
       END
```

```
T         SUBTYPE,FORTRAN,LMAP,LSTRAP
C      A ROUTINE TO PRINT OUT A MATRIX ARRAY
       SUBROUTINE PRNOUT(P,N)
       DIMENSION P(120,120)
       NR = N
       DO 100 I = 1,NR
       IB = 1
   20  IE = IB&7
       IF(IE-NR)28,28,24
   24  IE = NR
   28  IF(IB-1)36,36,60
   36  PRINT 44,I,(P(I,J),J = IB,IE)
   44  FORMAT(5H0 ROWI3,2X,1P8F15.6)
       GO TO 80
   60  PRINT 68,(P(I,J),J = IB,IE)
   68  FORMAT(1H 9X,1P8F15.6)
   80  IB = IE&1
       IF(IB-NR)20,20,100
  100  CONTINUE
       RETURN
       END
```

```
T         SUBTYPE,FORTRAN,LMAP,LSTRAP
C      THIS ROUTINE CALCULATES END POINTS AND SPACING FOR INEGRATION.
       SUBROUTINE CALENP
       COMMON DTR,RTD,CPI
       COMMON /CMVCOM/ NM,CMI(30),CMV,KMV,CM2,FM,QFM,TWM,PRODM
       COMMON /FNCCOM/ PNMLLG(61),BSSLSP(61),CNEUMN(61)
       COMMON /MTXCOM/ NRANK,NRANKI,CMTXRL(120,120),CMTXIM(120,120),SPRMT
      1X(120,120),CMXNRM(60)
       COMMON /THTCOM/ THETA,NTHETA,DLTHTA,SINTH,COSTH,ISMRL,ISWTCH(7),SR
      1MUL,SMULSS(7),CDH(6),DHM,NSECT,NDPS(6),EPPS(6),KSECT
       COMMON /BDYCOM/ CKR,DCKR,CKR2,CSKRX,SNKRX,CONK,BRXT,ALPHA,IBODY,QB
      1,SNALPH,CSALPH
       SNALPH = SIN(ALPHA)
       CSALPH = COS(ALPHA)
       QB = (1.0-BRXT)*(1.0-SNALPH)/2.0
C      CALCULATE THE FIRST END POINT AND STEP SIZE
       TANGAM = SNALPH*CSALPH/(QB-SNALPH*SNALPH)
       GAMMA = ATAN(TANGAM)
       IF(GAMMA)20,32,32
   20  GAMMA = GAMMA&CPI
   32  EPPS(1) = GAMMA
       CDVD = NDPS(1)
       CDH(1) = EPPS(1)/CDVD
C      CALCULATE THE SECOND END POINT AND STEP SIZE.
       TANPSI = -BRXT*SNALPH*CSALPH/(1.0-QB-BRXT*CSALPH*CSALPH)
       PSI = ATAN(TANPSI)
       IF(PSI)60,72,72
   60  PSI = PSI&CPI
   72  EPPS(2) = PSI
       CDVD = NDPS(2)
       CDH(2) = (EPPS(2)-EPPS(1))/CDVD
C      COMPUTE THIRD END POINT AND STEP SIZE.
       EPPS(3) = CPI
       CDVD = NDPS(3)
       CDH(3) = (EPPS(3)-EPPS(2))/CDVD
       RETURN
       END
```

```
T        SUBTYPE,FORTRAN,LMAP,LSTRAP
C      THIS ROUTINE COMPUTES KR AND ITS DERIVATIVE AS A FUNCTION OF THETA
       SUBROUTINE GENKR
       COMMON DTR,RTD,CPI
       COMMON /CMVCOM/ NM,CMI(30),CMV,KMV,CM2,EM,DEM,TWM,PRODM
       COMMON /FNCCOM/ PNMLLG(61),BSSLSP(61),CNEUMN(61)
       COMMON /MTXCOM/ NRANK,NRANKI,CMTXRL(120,120),CMTXIM(120,120),SPRMT
      1X(120,120),CMXNRM(60)
       COMMON /THTCOM/ THETA,NTHETA,DLTHTA,SINTH,COSTH,ISMRL,ISWTCH(7),SB
      1MUL,SMULSS(7),CDH(6),DHM,NSECT,NDPSI(6),EPPSI(6),KSECT
       COMMON /BDYCOM/ CKR,DCKR,CKR2,CSKRX,SNKRX,CONK,BRXT,ALPHA,IBODY,QB
      1,SNALPH,CSALPH
       KSECT = KSECT
C      DETERMINE SECTION FOR INTEGRATION
       IF(KSECT-2)40,140,240
C      SECTION 1
    40 QUAN1 = SQRT(1.0-(QB*SINTH/SNALPH)**2)
       CKR = CONK*(QB*COSTH/SNALPH&QUAN1)
       DCKR = -CONK*(QB*SINTH/SNALPH)*(1.0&QB*COSTH/(SNALPH*QUAN1))
       GO TO 300
C      SECTION 2
   140 QUAN2 = THETA-ALPHA
       SNON2 = SIN(QUAN2)
       CKR = CONK*(1.0-QB)/SNON2
       DCKR = -CONK*(1.0-QB)*COS(QUAN2)/(SNON2*SNON2)
       GO TO 300
C      SECTION 3
   240 QUAN3 = (1.0-BRXT-QB)/SNALPH
       QUNSQ = SQRT(BRXT*BRXT-(QUAN3*SINTH)**2)
       CKR = CONK*(QUNSQ-QUAN3*COSTH)
       DCKR = CONK*(QUAN3*SINTH-QUAN3*QUAN3*SINTH*COSTH/QUNSQ)
   300 RETURN
       END
```

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APPENDIX III

A NUMERICAL EXAMPLE: THE SPHERE-CONE-SPHERE

****

ELECTROMAGNETIC SCATTERING FROM GENERAL AXISYMMETRIC CONDUCTORS

****

		CASES	MATRIX RANK	SECTIONS	BODY SHAPE	U VECTOR
		4	6	3	9	46
	BODY PARAMETERS		K(A)	BETA/RHO	ALPHA	
			1.000	.794	15.000	
INTEGRATIONS/SECTION	64	64	64			
END POINTS	87.8907	132.6005	180.0000			

M = 0

REAL PART OF O1(NORMALIZED,TRANSPOSED)

ROW 1	-2.013760E-01	-5.250234E-03	-3.415709E-03	1.215186E-04	-1.278269E-05	5.076459E-07
ROW 2	-5.250234E-03	-1.983892E-02	-4.114353E-04	-2.500533E-04	5.430334F-06	-8.106394E-07
ROW 3	-3.415709E-03	-4.114353E-04	-8.399868E-04	-1.540606E-05	-7.985793E-06	1.120602E-07
ROW 4	1.215186E-04	-2.500533E-04	-1.540606E-05	-2.006002E-05	-3.313540E-07	-1.478181E-07
ROW 5	-1.278269E-05	5.430334E-06	-7.985793F-06	-3.313540E-07	-3.098136E-07	-4.608528F-09
ROW 6	5.076459E-07	-8.106394E-07	1.120602E-07	-1.478181E-07	-4.608528F-09	-3.357028E-09

IMAGINARY PART OF O1(NORMALIZED,TRANSPOSED)

ROW 1	8.482023E-01	-1.841377E-01	-2.083360E-01	4.156567E+00	-1.110176E+01	-1.441091E+02
ROW 2	-1.019649E-02	6.880517E-01	8.275353E-03	-5.163250E-01	2.262755E+00	3.443157E+00
ROW 3	2.980026E-03	-1.005275E-05	6.042122E-01	-3.370680E-02	-3.703436E-01	3.809894F+00
ROW 4	-1.429752E-04	1.121553E-03	-1.100414E-04	5.752027F-01	-4.184924E-02	-2.519765F-03
ROW 5	5.861538E-06	-3.128003E-05	3.529400E-04	-2.494046E-04	5.646671F-01	-7.883293F-02
ROW 6	-6.993570E-08	1.630544E-06	-9.989378E-06	2.110116E-04	-4.901102E-04	5.511401F-01

REAL PART OF O4(NORMALIZED,TRANSPOSED)

ROW 1	2.013760E-01	5.250234E-03	3.415709E-03	-1.215186E-04	1.278269E-05	-5.076459E-07
ROW 2	5.250234E-03	1.983892E-02	4.114353F-04	2.500533E-04	-5.430334E-06	8.106394E-07
ROW 3	3.415709E-03	4.114353E-04	8.399868E-04	1.540606E-05	7.985793F-06	-1.120602E-07
ROW 4	-1.215186E-04	2.500533E-04	1.540606F-05	2.006002E-05	3.313540E-07	1.478181F-07
ROW 5	1.278269E-05	-5.430334E-06	7.985793E-06	3.313540E-07	3.098136E-07	4.608528F-09
ROW 6	-5.076459E-07	8.106394E-07	-1.120602E-07	1.478181F-07	4.608528E-09	3.357028E-09

IMAGINARY PART OF O4(NORMALIZED,TRANSPOSED)

ROW 1	1.517977E-01	1.841377E-01	2.083360E-01	-4.156567E+00	1.110176E+01	1.441091F+02
ROW 2	1.019649E-02	3.119483E-01	-8.275353E-03	5.163250E-01	-2.262755E+00	-3.443157E+00
ROW 3	-2.980026E-03	1.005275E-05	3.957878E-01	3.370680E-02	3.703436F-01	-3.809894F+00
ROW 4	1.429752E-04	-1.121553E-03	1.100414E-04	4.247973E-01	4.184924E-02	2.519765F-03
ROW 5	-5.861538E-06	3.128003E-05	-3.529400E-04	2.494046E-04	4.353329E-01	7.883293F-02
ROW 6	6.993570E-08	-1.630544E-06	9.989378E-06	-2.110116E-04	4.901102E-04	4.488599E-01

REAL SECTION OF ORTHOGONALIZED $\hat{O}$ MATRIX.

ROW 1	-2.351224E-01	-.000000E 00	-1.103360E-02	.000000E 00	-4.340343E-03	-.000000E 00	1.609859E-04	.000000E 00
	-1.707707E-05	-.000000E 00	7.225856E-07	-.000000E 00				
ROW 2	.000000E 00	7.935493E-01	.000000E 00	-1.635941E-02	.000000E 00	1.117055E-02	.000000E 00	-5.098768E-04
	.000000E 00	3.966673E-05	.000000E 00	-1.459251E-06				
ROW 3	-7.331480E-03	-.000000E 00	-2.911157E-02	.000000E 00	-5.679232E-04	-.000000E 00	-3.821475E-04	.000000E 00
	8.428304E-06	-.000000E 00	-1.254648E-06	-.000000E 00				
ROW 4	.000000E 00	1.765869E-02	.000000E 00	6.211844E-02	.000000E 00	1.414359E-03	.000000E 00	7.374121E-04
	.000000E 00	-1.543721E-05	.000000E 00	2.321461E-06				
ROW 5	-5.657444E-03	.000000E 00	-6.910358E-04	-.000000E 00	-1.400975E-03	.000000E 00	-2.706441E-05	-.000000E 00
	-1.334320E-05	.000000E 00	1.821811E-07	.000000E 00				
ROW 6	-.000000E 00	8.605696E-03	-.000000E 00	1.020580E-03	-.000000E 00	2.097602E-03	-.000000E 00	3.578012E-05
	-.000000E 00	1.989545E-05	-.000000E 00	-2.901949E-07				
ROW 7	2.096244E-04	.000000E 00	-4.340501E-04	-.000000E 00	-2.781612E-05	.000000E 00	-3.492239E-05	-.000000E 00
	-6.208722E-07	.000000E 00	-2.576996E-07	.000000E 00				
ROW 8	-.000000E 00	-2.890061E-04	-.000000E 00	5.899358E-04	-.000000E 00	3.444625E-05	-.000000E 00	4.715394E-05
	-.000000E 00	7.016803E-07	-.000000E 00	3.468885E-07				
ROW 9	-2.251095E-05	-.000000E 00	9.411395E-06	.000000E 00	-1.411567E-05	-.000000E 00	-6.245729E-07	.000000E 00
	-5.499094E-07	-.000000E 00	-8.582602E-09	-.000000E 00				
ROW 10	.000000E 00	2.957491E-05	.000000E 00	-1.280553E-05	.000000E 00	1.839323E-05	.000000E 00	7.012946E-07
	.000000E 00	7.099412E-07	.000000E 00	9.919662E-09				
ROW 11	9.210829E-07	.000000E 00	-1.470840E-06	-.000000E 00	2.033243E-07	.000000E 00	-2.682061E-07	-.000000E 00
	-8.361805E-09	.000000E 00	-6.091059E-09	.000000E 00				
ROW 12	.000000E 00	-1.130967E-06	.000000E 00	1.805995E-06	.000000E 00	-2.496550E-07	.000000E 00	3.293187E-07
	.000000E 00	1.026718E-08	.000000E 00	7.479006E-09				

IMAGINARY SECTION OF ORTHOGONALIZED $\hat{Q}$ MATRIX.

ROW 1	9.717925E-01	-.000000E 00	1.259013E-02	.000000E 00	-6.127979E-03	-.000000E 00	2.598185E-04	.000000E 00
	-1.071368E-05	.000000E 00	6.725821E-08	.000000E 00				
ROW 2	-.000000E 00	6.073229E-01	-.000000E 00	-3.225749E-02	-.000000E 00	-2.255700E-03	-.000000E 00	-5.124876E-05
	-.000000E 00	-1.515172E-05	-.000000E 00	7.698270E-07				
ROW 3	-1.505475E-02	-.000000E 00	9.994337E-01	.000000E 00	-3.327090E-05	-.000000E 00	-1.959666E-03	.000000E 00
	5.447474E-05	.000000E 00	-2.196656E-06	.000000E 00				
ROW 4	-.000000E 00	3.155083E-02	-.000000E 00	9.974090E-01	-.000000E 00	-1.138229E-04	-.000000E 00	2.598387E-03
	-.000000E 00	-7.346793E-05	-.000000E 00	4.830472E-06				
ROW 5	4.922130E-03	.000000E 00	4.532070E-05	-.000000E 00	9.999705E-01	.000000E 00	1.480664E-04	-.000000E 00
	-6.227010E-04	-.000000E 00	1.751893E-05	-.000000E 00				
ROW 6	.000000E 00	-7.534516E-03	.000000E 00	1.344577E-04	.000000E 00	9.999315E-01	.000000E 00	-3.344752E-04
	.000000E 00	8.146999E-04	.000000E 00	-2.329022E-05				
ROW 7	-2.478133E-04	.000000E 00	1.945878E-03	-.000000E 00	-1.458441E-04	.000000E 00	9.999978E-01	-.000000E 00
	3.884890E-04	-.000000E 00	-3.825268E-04	-.000000E 00				
ROW 8	.000000E 00	3.379299E-04	.000000E 00	-2.647536E-03	.000000E 00	3.397571E-04	.000000E 00	9.999958E-01
	.000000E 00	-6.576739E-04	.000000E 00	4.708033E-04				
ROW 9	1.036398E-05	-.000000E 00	-5.498669E-05	.000000E 00	6.225717E-04	-.000000E 00	-3.879406E-04	.000000E 00
	9.999993E-01	.000000E 00	8.894256E-04	.000000E 00				
ROW 10	-.000000E 00	-1.349555E-05	-.000000E 00	7.252926E-05	-.000000E 00	-8.149514E-04	-.000000E 00	6.586787E-04
	-.000000E 00	9.999989E-01	-.000000E 00	-1.091571E-03				
ROW 11	-1.268927E-07	.000000E 00	2.958492E-06	-.000000E 00	-1.812493E-05	.000000E 00	3.828637E-04	-.000000E 00
	-8.892657E-04	-.000000E 00	9.999995E-01	-.000000E 00				
ROW 12	.000000E 00	1.558073E-07	.000000E 00	-3.632632E-06	.000000E 00	2.225498E-05	.000000E 00	-4.701054E-04
	.000000E 00	1.091899E-03	.000000E 00	9.999993E-01				

MATRIX T(1),REAL

ROW 1	5.536834E-02	2.811496E-03	1.032596E-03	-3.490388E-05	4.028779E-06	-1.617821E-07
ROW 2	2.811496E-03	9.698898E-04	6.540281E-05	9.382515E-06	-4.745470E-08	2.853795E-08
ROW 3	1.032596E-03	6.540281E-05	2.112482E-05	-4.428070E-07	8.805222E-08	-2.671668E-09
ROW 4	-3.490388E-05	9.382515E-06	-4.428070E-07	1.739057E-07	-5.586869E-09	5.998628E-10
ROW 5	4.028779E-06	-4.745470E-08	8.805222E-08	-5.586869E-09	5.413915E-10	-2.518031E-11
ROW 6	-1.617821E-07	2.853795E-08	-2.671668E-09	5.998628E-10	-2.518031E-11	2.195983E-12

MATRIX T(1),IMAGINARY

ROW 1	2.284077E-01	1.028740E-02	4.216252E-03	-1.620735E-04	1.678778E-05	-7.220522E-07
ROW 2	1.028740E-02	2.923487E-02	6.223639E-04	3.799735E-04	-8.206747E-06	1.245333E-06
ROW 3	4.216252E-03	6.223639E-04	1.374322E-03	2.803270E-05	1.323869E-05	-1.778218E-07
ROW 4	-1.620735E-04	3.799735E-04	2.803270E-05	3.413538E-05	6.435900E-07	2.550246E-07
ROW 5	1.678778E-05	-8.206747E-06	1.323869E-05	6.435900E-07	5.411919E-07	8.866825E-09
ROW 6	-7.220522E-07	1.245333E-06	-1.778218E-07	2.550246E-07	8.866825E-09	5.994117E-09

MATRIX T(4),REAL

ROW 1	6.301065E-01	-1.187646E-02	8.907403E-03	-3.912963E-04	3.137594E-05	-1.119591E-06
ROW 2	-1.187646E-02	4.127721E-03	-9.272504E-05	5.421250E-05	-1.587150E-06	1.679864E-07
ROW 3	8.907403E-03	-9.272504E-05	1.311831E-04	-4.575951E-06	4.630356E-07	-1.361384E-08
ROW 4	-3.912963E-04	5.421250E-05	-4.575951E-06	8.072552E-07	-3.086328E-08	2.461895E-09
ROW 5	3.137594E-05	-1.587150E-06	4.630356E-07	-3.086328E-08	2.208583E-09	-9.924363E-11
ROW 6	-1.119591E-06	1.679864E-07	-1.361384E-08	2.461895E-09	-9.924363E-11	7.723291E-12

MATRIX T(4),IMAGINARY

ROW 1	-4.824329E-01	7.983048E-03	-6.812965E-03	2.866476E-04	-2.345379E-05	8.106888E-07
ROW 2	7.983048E-03	-6.248378E-02	-1.050553E-03	-7.518288E-04	1.667590E-05	-2.361561E-06
ROW 3	-6.812965E-03	-1.050553E-03	-2.072097E-03	-3.685932E-05	-1.980603E-05	2.870377E-07
ROW 4	2.866476E-04	-7.518288E-04	-3.685932E-05	-4.908430E-05	-6.533409E-07	-3.530940E-07
ROW 5	-2.345379E-05	1.667590E-05	-1.980603E-05	-6.533409E-07	-7.262320E-07	-9.314813E-09
ROW 6	8.106888E-07	-2.361561E-06	2.870377E-07	-3.530940E-07	-9.314813E-09	-7.648351E-09

********** ACCUMULATED SUMS FOR M = 0 **********

CLASS 1 ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	9.000000E+01
4.00	1.709006E-02	-2.201233E-02	-1.709006E-02	-3.108635E-03	-7.266870E-03	6.247100E-05	-1.131604E+02
8.00	6.812894E-02	-8.716015E-02	-6.812894E-02	-1.258449E-02	-2.903832E-02	1.001594E-03	-1.134307E+02
12.00	1.524199E-01	-1.928329E-01	-1.524199E-01	-2.886441E-02	-6.521954E-02	5.086743E-03	-1.138729E+02
16.00	2.687924E-01	-3.348647E-01	-2.687924E-01	-5.263302E-02	-1.156287E-01	1.614024E-02	-1.144745E+02
20.00	4.155903E-01	-5.077949E-01	-4.155903E-01	-8.475792E-02	-1.799636E-01	3.957380E-02	-1.152192E+02
24.00	5.906593E-01	-7.052019E-01	-5.906593E-01	-1.261999E-01	-2.577589E-01	8.236607E-02	-1.160866E+02
28.00	7.913357E-01	-9.200811E-01	-7.913357E-01	-1.779006E-01	-3.483390E-01	1.529887E-01	-1.170539E+02
32.00	1.014443E+00	-1.145239E+00	-1.014443E+00	-2.406506E-01	-4.507702E-01	2.611065E-01	-1.180962E+02
36.00	1.256296E+00	-1.373670E+00	-1.256296E+00	-3.149462E-01	-5.638187E-01	4.170826E-01	-1.191875E+02
40.00	1.512725E+00	-1.598899E+00	-1.512725E+00	-4.008417E-01	-6.859188E-01	6.311587E-01	-1.203014E+02
44.00	1.779111E+00	-1.815250E+00	-1.779111E+00	-4.978125E-01	-8.151572E-01	9.122985E-01	-1.214122E+02
48.00	2.050448E+00	-2.018045E+00	-2.050448E+00	-6.046408E-01	-9.492770E-01	1.265717E+00	-1.224951E+02
52.00	2.321423E+00	-2.203718E+00	-2.321423E+00	-7.193425E-01	-1.085705E+00	1.696208E+00	-1.235268E+02
56.00	2.586519E+00	-2.369835E+00	-2.586519E+00	-8.391482E-01	-1.221601E+00	2.196479E+00	-1.244861E+02
60.00	2.840142E+00	-2.515037E+00	-2.840142E+00	-9.605538E-01	-1.353937E+00	2.755809E+00	-1.253539E+02
64.00	3.076761E+00	-2.638918E+00	-3.076761E+00	-1.079444E+00	-1.479585E+00	3.354372E+00	-1.261129E+02
68.00	3.291062E+00	-2.741841E+00	-3.291062E+00	-1.191294E+00	-1.595434E+00	3.964591E+00	-1.267483E+02
72.00	3.478104E+00	-2.824721E+00	-3.478104E+00	-1.291430E+00	-1.698504E+00	4.552707E+00	-1.272470E+02
76.00	3.633480E+00	-2.888791E+00	-3.633480E+00	-1.375342E+00	-1.786070E+00	5.081611E+00	-1.275976E+02
80.00	3.753462E+00	-2.935373E+00	-3.753462E+00	-1.439010E+00	-1.855775E+00	5.514650E+00	-1.277908E+02
84.00	3.835129E+00	-2.965666E+00	-3.835129E+00	-1.479222E+00	-1.905727E+00	5.819890E+00	-1.278186E+02
88.00	3.876475E+00	-2.980571E+00	-3.876475E+00	-1.493833E+00	-1.934579E+00	5.974132E+00	-1.276744E+02
92.00	3.876475E+00	-2.980571E+00	-3.876475E+00	-1.491963E+00	-1.941580E+00	5.965946E+00	-1.273536E+02
96.00	3.835129E+00	-2.965666E+00	-3.835129E+00	-1.444072E+00	-1.926598E+00	5.797126E+00	-1.268532E+02
100.00	3.753462E+00	-2.935373E+00	-3.753462E+00	-1.381945E+00	-1.890114E+00	5.482302E+00	-1.261722E+02
104.00	3.633480E+00	-2.888791E+00	-3.633480E+00	-1.298546E+00	-1.833186E+00	5.044793E+00	-1.253120E+02
108.00	3.478104E+00	-2.824721E+00	-3.478104E+00	-1.197793E+00	-1.757395E+00	4.523147E+00	-1.242773E+02
112.00	3.291062E+00	-2.741841E+00	-3.291062E+00	-1.084252E+00	-1.664770E+00	3.947061E+00	-1.230759E+02
116.00	3.076761E+00	-2.638918E+00	-3.076761E+00	-9.627994E-01	-1.557697E+00	3.353402E+00	-1.217199E+02
120.00	2.840142E+00	-2.515037E+00	-2.840142E+00	-8.382822E-01	-1.438830E+00	2.772949E+00	-1.202257E+02
124.00	2.586519E+00	-2.369835E+00	-2.586519E+00	-7.152081E-01	-1.310996E+00	2.230233E+00	-1.186145E+02
128.00	2.321423E+00	-2.203718E+00	-2.321423E+00	-5.974930E-01	-1.177104E+00	1.742573E+00	-1.169171E+02
132.00	2.050448E+00	-2.018045E+00	-2.050448E+00	-4.882830E-01	-1.040069E+00	1.320163E+00	-1.151487E+02
136.00	1.779111E+00	-1.815250E+00	-1.779111E+00	-3.898579E-01	-9.027351E-01	9.669198E-01	-1.133577E+02
140.00	1.512725E+00	-1.598899E+00	-1.512725E+00	-3.036187E-01	-7.678249E-01	6.817364E-01	-1.115749E+02
144.00	1.256296E+00	-1.373670E+00	-1.256296E+00	-2.301159E-01	-6.378893E-01	4.598561E-01	-1.098367E+02
148.00	1.014443E+00	-1.145239E+00	-1.014443E+00	-1.692061E-01	-5.152753E-01	2.941394E-01	-1.081791E+02
152.00	7.913357E-01	-9.200811E-01	-7.913357E-01	-1.201475E-01	-4.021019E-01	1.761214E-01	-1.066361E+02
156.00	5.906593E-01	-7.052019E-01	-5.906593E-01	-8.178951E-02	-3.002452E-01	9.683669E-02	-1.052381E+02
160.00	4.155903E-01	-5.077949E-01	-4.155903E-01	-5.273768E-02	-2.113301E-01	4.744169E-02	-1.040129E+02
164.00	2.687924E-01	-3.348647E-01	-2.687924E-01	-3.151574E-02	-1.367274E-01	1.968763E-02	-1.029800E+02
168.00	1.524199E-01	-1.928329E-01	-1.524199E-01	-1.671065E-02	-7.755343E-02	6.293781E-03	-1.021598E+02
172.00	6.812894E-02	-8.716015E-02	-6.812894E-02	-7.094955E-03	-3.467232E-02	1.252508E-03	-1.015647E+02
176.00	1.709006E-02	-2.201233E-02	-1.709006E-02	-1.723024E-03	-8.698659E-03	7.863548E-05	-1.012041E+02
180.00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	9.000000E+01

********** ACCUMULATED SUMS FOR M = 0 **********

CLASS 2 ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	9.000000E+01
4.00	1.536854E-03	-1.030375E-02	1.536854E-03	-1.315516E-03	7.725338E-05	1.736551E-06	1.766392E+02
8.00	6.125075E-03	-4.079342E-02	6.125075E-03	-5.333131E-03	3.282463E-04	2.855003E-05	1.764780E+02
12.00	1.369754E-02	-9.023346E-02	1.369754E-02	-1.226121E-02	8.096921E-04	1.509928E-04	1.762218E+02
16.00	2.414208E-02	-1.566592E-01	2.414208E-02	-2.243067E-02	1.612853E-03	5.057364E-04	1.758873E+02
20.00	3.730116E-02	-2.375086E-01	3.730116E-02	-3.626986E-02	2.858730E-03	1.323675E-03	1.754934E+02
24.00	5.297166E-02	-3.297886E-01	5.297166E-02	-5.426818E-02	4.691483E-03	2.967045E-03	1.750591E+02
28.00	7.090496E-02	-4.302601E-01	7.090496E-02	-7.692842E-02	7.270226E-03	5.970837E-03	1.746012E+02
32.00	9.080758E-02	-5.356253E-01	9.080758E-02	-1.047080E-01	1.075937E-02	1.107954E-02	1.741331E+02
36.00	1.123427E-01	-6.427016E-01	1.123427E-01	-1.379513E-01	1.531778E-02	1.926520E-02	1.736640E+02
40.00	1.351328E-01	-7.485702E-01	1.351328E-01	-1.768160E-01	2.108690E-02	3.170856E-02	1.731991E+02
44.00	1.587637E-01	-8.506890E-01	1.587637E-01	-2.211998E-01	2.817845E-02	4.972338E-02	1.727402E+02
48.00	1.827900E-01	-9.469659E-01	1.827900E-01	-2.706743E-01	3.666195E-02	7.460869E-02	1.722864E+02
52.00	2.067426E-01	-1.035791E+00	2.067426E-01	-3.244352E-01	4.655281E-02	1.074254E-01	1.718344E+02
56.00	2.301375E-01	-1.116031E+00	2.301375E-01	-3.812774E-01	5.780184E-02	1.487135E-01	1.713296E+02
60.00	2.524867E-01	-1.186993E+00	2.524867E-01	-4.396042E-01	7.028708E-02	1.981922E-01	1.709160E+02
64.00	2.733095E-01	-1.248360E+00	2.733095E-01	-4.974753E-01	8.380888E-02	2.545056E-01	1.704373E+02
68.00	2.921462E-01	-1.300115E+00	2.921462E-01	-5.526961E-01	9.808946E-02	3.150945E-01	1.699362E+02
72.00	3.085703E-01	-1.342453E+00	3.085703E-01	-6.029450E-01	1.127776E-01	3.762614E-01	1.694056E+02
76.00	3.222026E-01	-1.375695E+00	3.222026E-01	-6.459290E-01	1.274591E-01	4.334701E-01	1.688374E+02
80.00	3.327228E-01	-1.400207E+00	3.327228E-01	-6.795552E-01	1.416731E 1	4.818665E-01	1.682237E+02
84.00	3.398802E-01	-1.416329E+00	3.398802E-01	-7.020988E-0.	1.549334E-01	5.169471E-01	1.675559E+02
88.00	3.435027E-01	-1.424321E+00	3.435027E-01	-7.123508E-01	1.667537E-01	5.352505E-01	1.668249E+02
92.00	3.435027E-01	-1.424321E+00	3.435027E-01	-7.097253E-01	1.766756E-01	5.349242E-01	1.660212E+02
96.00	3.398802E-01	-1.416329E+00	3.398802E-01	-6.943155E-01	1.842957E-01	5.160388E-01	1.651345E+02
100.00	3.327228E-01	-1.400207E+00	3.327228E-01	-6.668892E-01	1.892904E-01	4.805721E-01	1.641539E+02
104.00	3.222026E-01	-1.375695E+00	3.222026E-01	-6.288251E-01	1.914368E-01	4.323690E-01	1.630679E+02
108.00	3.085703E-01	-1.342453E+00	3.085703E-01	-5.819966E-01	1.906253E-01	3.759581E-01	1.618645E+02
112.00	2.921462E-01	-1.300115E+00	2.921462E-01	-5.286185E-01	1.868663E-01	3.143566E-01	1.605315E+02
116.00	2.733095E-01	-1.248360E+00	2.733095E-01	-4.710731E-01	1.802878E-01	2.544135E-01	1.590573E+02
120.00	2.524867E-01	-1.186993E+00	2.524867E-01	-4.117351E-01	1.711257E-01	1.988098E-01	1.574312E+02
124.00	2.301375E-01	-1.116031E+00	2.301375E-01	-3.528145E-01	1.597086E-01	1.499849E-01	1.556452E+02
128.00	2.067426E-01	-1.035791E+00	2.067426E-01	-2.962293E-01	1.464372E-01	1.091956E-01	1.536951E+02
132.00	1.827900E-01	-9.469659E-01	1.827900E-01	-2.435178E-01	1.317623E-01	7.666221E-02	1.515831E+02
136.00	1.587637E-01	-8.506890E-01	1.587637E-01	-1.957945E-01	1.161621E-01	5.182912E-02	1.493199E+02
140.00	1.351328E-01	-7.485702E-01	1.351328E-01	-1.537459E-01	1.001208E-01	3.366197E-02	1.469274E+02
144.00	1.123427E-01	-6.427016E-01	1.123427E-01	-1.176611E-01	8.411039E-02	2.091869E-02	1.444408E+02
148.00	9.080758E-02	-5.356253E-01	9.080758E-02	-8.748860E-02	6.857542E-02	1.235684E-02	1.419099E+02
152.00	7.090496E-02	-4.302601E-01	7.090496E-02	-6.290960E-02	5.392214E-02	6.865215E-03	1.393989E+02
156.00	5.297166E-02	-3.297886E-01	5.297166E-02	-4.341837E-02	4.051110E-02	3.526304E-03	1.369839E+02
160.00	3.730116E-02	-2.375086E-01	3.730116E-02	-2.840257E-02	2.865310E-02	1.627706E-03	1.347484E+02
164.00	2.414208E-02	-1.566592E-01	2.414208E-02	-1.721731E-02	1.860774E-02	6.426835E-04	1.327274E+02
168.00	1.369754E-02	-9.023346E-02	1.369754E-02	-9.249243E-03	1.058380E-02	1.975653E-04	1.311504E+02
172.00	6.125075E-03	-4.079342E-02	6.125075E-03	-3.968920E-03	4.740761E-03	3.822714E-05	1.299357E+02
176.00	1.536854E-03	-1.030375E-02	1.536854E-03	-9.705952E-04	1.190689E-03	2.359795E-06	1.291853E+02
180.00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	.000000E 00	9.000000E+01

M = 1

REAL PART OF Q1(NORMALIZED,TRANSPOSED)

ROW 1	-1.054067E-01	-5.438268E-03	-2.636357E-03	8.777420E-05	-9.360268E-06	3.741119E-07
ROW 2	-5.438268E-03	-1.787653E-02	-4.870747E-04	-2.146196E-04	4.218730E-06	-6.783886E-07
ROW 3	-2.636357E-03	-4.870747E-04	-7.670009E-04	-1.763342E-05	-7.160156E-06	8.872623E-08
ROW 4	8.777420E-05	-2.146196E-04	-1.763342E-05	-1.857069E-05	-3.670562E-07	-1.358714E-07
ROW 5	-9.360268E-06	4.218730E-06	-7.160156E-06	-3.670562E-07	-2.899052E-07	-5.016308E-09
ROW 6	3.741119E-07	-6.783886E-07	8.872623E-08	-1.358714E-07	-5.016308E-09	-3.166759E-09

IMAGINARY PART OF Q1(NORMALIZED,TRANSPOSED)

ROW 1	6.816966E-01	1.106342E-01	1.310632E-01	-2.730311E+00	6.789824E+00	1.097974E+02
ROW 2	1.020002E-02	6.677350E-01	-1.216168E-01	2.024308E-01	2.539579E+00	-1.660960E+01
ROW 3	1.618872E-03	-7.041528E-04	6.103028E-01	-5.263086E-02	-1.797586E-01	3.475714E+00
ROW 4	-1.030677E-04	1.105948E-03	-5.978075E-04	5.785184E-01	-4.141653E-02	-6.234960E-01
ROW 5	4.419619E-06	-4.170095E-05	4.224279E-04	-2.669728E-04	5.575038E-01	-1.005096E-02
ROW 6	-2.566840E-07	2.854249E-06	-1.160699E-05	1.532519E-04	5.462542E-05	5.510089E-01

REAL PART OF Q2(NORMALIZED,TRANSPOSED)

ROW 1	1.433912E-02	1.091577E-02	-4.638819E-04	6.267305E-05	-2.906507E-06	1.590691E-07
ROW 2	1.091577E-02	-1.707472E-04	3.652389E-04	-1.209629E-05	1.772525E-06	-6.377933E-08
ROW 3	-4.638819E-04	3.652389E-04	-8.202676E-06	7.079765E-06	-1.822518E-07	2.821271E-08
ROW 4	6.267305E-05	-1.209629E-05	7.079765E-06	-1.380688E-07	9.040970E-08	-1.930203E-09
ROW 5	-2.906507E-06	1.772525E-06	-1.822518E-07	9.040970E-08	-1.541303E-09	8.417351E-10
ROW 6	1.590691E-07	-6.377933E-08	2.821271E-08	-1.930203E-09	8.417351E-10	-1.289847E-11

IMAGINARY PART OF Q2(NORMALIZED,TRANSPOSED)

ROW 1	-1.734398E-02	-1.870584E-02	-1.473073E-01	-3.425618E-01	1.011777E+01	-3.442396E+01
ROW 2	-1.870584E-02	3.717232E-03	-3.735276E-03	-5.760589E-02	-5.613637E-01	9.873763E+00
ROW 3	1.236915E-03	-3.735276E-03	1.017702E-03	-9.517707E-04	-3.957547E-02	-5.560416E-01
ROW 4	-8.437283E-05	1.831521E-04	-9.517729E-04	3.827018E-04	-3.442227E-04	-3.132514E-02
ROW 5	3.627322E-06	-1.222689E-05	3.873739E-05	-3.442270E-04	1.980326E-04	-1.645263E-04
ROW 6	-1.446869E-07	3.366739E-07	-1.890784E-06	1.253497E-05	-1.645449E-04	1.411674E-04

REAL SECTION OF ORTHOGONALIZED $\hat{Q}$ MATRIX.

ROW 1	-2.556708E-01	6.064090E-02	-1.794013E-03	2.308473E-02	-3.431410E-03	2.013550E-04	1.115804E-04	1.535408E-04
	-1.150477E-05	-5.179369E-06	4.640876E-07	4.590836E-07				
ROW 2	4.553357E-02	5.143513E-01	2.732876E-02	2.779284E-02	-1.364040E-03	8.091135E-03	1.107696E-04	-2.528830E-04
	-3.962721E-06	2.941896E-05	1.643013E-07	-1.143389E-06				
ROW 3	-9.555576E-03	1.604915E-02	-2.682318E-02	-1.125819E-03	-9.253175E-04	5.762302E-04	-3.188628E-04	-2.740147E-05
	5.768749E-06	3.607543E-06	-1.011561E-06	-1.526496E-07				
ROW 4	3.311415E-02	1.395616E-02	-8.632702E-04	5.338013E-02	1.054277E-03	9.091897E-04	-4.322435E-05	6.474490E-04
	4.581299E-06	-1.458891E-05	-1.713622E-07	2.053624E-06				
ROW 5	-4.308063E-03	-7.835841E-04	-8.258080E-04	5.957262E-04	-1.263680E-03	-2.185610E-05	-3.096682E-05	1.206803E-05
	-1.197829E-05	-3.650307E-07	1.493331E-07	5.341363E-08				
ROW 6	-1.203616E-03	6.772214E-03	9.352757E-04	1.199568E-03	-2.373257E-05	1.952586E-03	1.745117E-05	4.106516E-05
	-4.906487E-07	1.786141E-05	6.530784E-08	-2.211806E-07				
ROW 7	1.509775E-04	1.085631E-04	-3.716199E-04	-2.153998E-05	-3.124640E-05	1.221824E-05	-3.240448E-05	-3.215195E-07
	-6.536457E-07	1.617686E-07	-2.388260E-07	-3.717397E-09				
ROW 8	1.487919E-04	-2.090044E-04	-2.895820E-05	5.076797E-04	1.673780E-05	4.051122E-05	-3.603822E-07	4.351166E-05
	2.068709E-07	8.387949E-07	-4.729290E-09	3.153530E-07				
ROW 9	-1.677187E-05	-5.225850E-06	7.541767E-06	3.186016E-06	-1.283996E-05	-3.391584E-07	-6.629795E-07	1.616417E-07
	-5.201639E-07	-3.191767E-09	-8.848751E-09	1.548091E-09				
ROW 10	-6.575954E-06	2.117227E-05	4.008101E-06	-9.568113E-06	-4.131810E-07	1.618572E-05	2.042467E-07	8.226702E-07
	-3.531860E-09	6.549047E-07	1.854393E-09	1.156908E-08				
ROW 11	6.787559E-07	2.891625E-07	-1.231094E-06	-1.166122E-07	1.609892E-07	5.131467E-08	-2.465842E-07	-3.675711E-09
	-9.104931E-09	1.521250E-09	-5.747184E-09	-2.487096E-11				
ROW 12	3.542812E-07	-8.332277E-07	-1.420503E-07	1.510917E-06	6.283576E-08	-1.976124E-07	-4.298978E-09	3.026149E-07
	1.874725E-09	1.117240E-08	-2.872766E-11	7.053053E-09				

IMAGINARY SECTION OF ORTHOGONALIZED $\hat{G}$ MATRIX.

ROW 1	9.613584E-01	4.313218E-02	-1.759228E-02	6.325043E-02	-3.328929E-03	-3.922976E-03	2.129200E-04	3.927559E-04
	-8.754666E-06	-2.018028E-05	5.116091E-07	1.016281E-06				
ROW 2	-5.964336E-02	8.531150E-01	1.726548E-02	1.100914E-02	-8.907735E-04	3.945408E-04	2.499533E-05	-1.147572E-04
	1.791372E-07	-1.561526E-06	-6.484116E-08	-1.277299E-07				
ROW 3	1.690407E-02	-2.712683E-02	9.988031E-01	-1.440288E-02	8.919654E-04	9.369307E-03	-1.899298E-03	-4.689460E-04
	7.193240E-05	3.589443E-05	-4.601816E-06	-1.118039E-06				
ROW 4	-5.728007E-02	-2.904301E-02	1.468243E-02	9.957228E-01	6.430321E-03	-1.976983E-03	-3.606147E-04	2.589206E-03
	2.077508E-05	-9.729819E-05	-6.032813E-07	7.203993E-06				
ROW 5	2.626276E-03	2.035974E-03	-9.917290E-04	-6.120812E-03	9.999569E-01	-2.874433E-03	1.000597E-03	2.748640E-03
	-7.555358E-04	-9.007686E-05	2.078460E-05	4.691954E-06				
ROW 6	3.191871E-03	-4.174865E-03	-9.612541E-03	2.054588E-03	2.879273E-03	9.999041E-01	1.664071E-03	-1.456906E-03
	-7.271431E-05	9.564048E-04	3.410645E-06	-2.626301E-05				
ROW 7	-1.777275E-04	-1.459929E-04	1.910681E-03	3.195629E-04	-9.977497E-04	-1.645995E-03	9.999952E-01	-9.633522E-04
	4.740075E-04	7.773539E-04	-2.779516E-04	-2.785209E-05				
ROW 8	-2.004539E-04	2.445570E-04	4.362921E-04	-2.622683E-03	-2.260453E-03	1.473852E-03	9.637070E-04	9.999917E-01
	6.193808E-04	-6.087413E-04	-2.339090E-05	3.413897E-04				
ROW 9	7.912325E-06	6.510076E-06	-7.468361E-05	-2.200072E-05	7.572605E-04	7.020856E-05	-4.732388E-04	-6.177508E-04
	9.999992E-01	-4.558855E-04	-9.920470E-05	3.662561E-04				
ROW 10	8.204342E-06	-1.000098E-05	-2.764496E-05	9.438391E-05	8.762342E-05	-9.552302E-04	-7.783602E-04	6.110581E-04
	4.558802E-04	9.999989E-01	2.987493E-04	1.219916E-04				
ROW 11	-4.656598E-07	-2.629117E-07	5.179614E-06	6.146410E-07	-2.106258E-05	-3.446245E-06	2.781137E-04	2.294389E-05
	9.934622E-05	-2.985554E-04	9.999999E-01	-3.144101E-04				
ROW 12	-3.222489E-07	5.716905E-07	7.498452E-07	-6.357027E-06	-4.211182E-06	2.585127E-05	2.791808E-05	-3.413249E-04
	-3.664769E-04	-1.216626E-04	3.144102E-04	9.999998E-01				

MATRIX T(1).REAL

ROW	1	6.864879E-02	1.933149E-03	8.644259E-04	-2.176106E-05	2.909714E-06	-1.079395E-07
ROW	2	1.933149E-03	1.472004E-03	-6.179309E-06	1.147119E-05	-2.366818E-07	3.096540E-08
ROW	3	8.644259E-04	-6.179309E-06	1.720177E-05	-2.447604E-07	5.955387E-08	-1.244000E-09
ROW	4	-2.176106E-05	1.147119E-05	-2.447604E-07	1.305762E-07	-3.376299E-09	4.042027E-10
ROW	5	2.909714E-06	-2.366818E-07	5.955387E-08	-3.376299E-09	3.467906E-10	-1.427183E-11
ROW	6	-1.079395E-07	3.096540E-08	-1.244000E-09	4.042027E-10	-1.427183E-11	1.378734E-12

MATRIX T(1),IMAGINARY

ROW	1	2.505806E-01	3.757753E-03	3.296882E-03	-9.772806E-05	1.102171E-05	-4.297151E-07
ROW	2	3.757753E-03	2.630924E-02	8.704852E-04	3.193654E-04	-5.978562E-06	1.019428E-06
ROW	3	3.296882E-03	8.704852E-04	1.245118E-03	3.191505E-05	1.190297E-05	-1.460618E-07
ROW	4	-9.772806E-05	3.193654E-04	3.191505E-05	3.175881E-05	6.811560E-07	2.364842E-07
ROW	5	1.102171E-05	-5.978562E-06	1.190297E-05	6.811560E-07	5.106525E-07	9.162527E-09
ROW	6	-4.297151E-07	1.019428E-06	-1.460618E-07	2.364842E-07	9.162527E-09	5.670956E-09

MATRIX T(2),REAL

ROW	1	8.220133E-03	-2.862128E-03	3.392902E-04	-2.916401E-05	2.126412E-06	-9.989183E-08
ROW	2	1.351220E-02	7.028694E-04	2.063568E-04	-6.953055E-06	7.460290E-07	-3.000796E-08
ROW	3	-9.089939E-04	-6.057669E-05	-1.132065E-05	5.105382E-07	-4.102885E-08	2.233785E-09
ROW	4	5.815910E-05	3.709062E-06	7.300262E-07	-2.979013E-08	2.478693E-09	-1.219261E-10
ROW	5	-2.573418E-06	-1.452709E-07	-2.758578E-08	1.888160E-09	-1.073409E-10	7.312503E-12
ROW	6	9.432542E-08	7.441239E-09	8.053158E-10	-4.916747E-11	2.351240E-12	-1.793687E-13

MATRIX T(2),IMAGINARY

ROW	1	-2.711147E-02	-1.746369E-02	3.251811E-04	-1.252955E-04	5.781378E-06	-3.886992E-07
ROW	2	-2.398436E-02	2.789191E-04	-7.063376E-04	2.531820E-05	-3.817400E-06	1.479290E-07
ROW	3	1.319671E-03	-8.386725E-04	1.785304E-05	-1.594091E-05	4.151471E-07	-6.462707E-08
ROW	4	-1.090857E-04	2.995392E-05	-1.429498E-05	3.549493E-07	-1.894678E-07	4.119105E-09
ROW	5	4.188621E-06	-3.778878E-06	3.662616E-07	-1.866896E-07	3.316896E-09	-1.747525E-09
ROW	6	-1.926674E-07	1.257407E-07	-5.436146E-08	4.057898E-09	-1.697122E-09	2.556549E-11

MATRIX T(3),REAL

ROW 1	8.220133E-03	1.351220E-02	-9.089939E-04	5.815910E-05	-2.573418E-06	9.432542E-08
ROW 2	-2.862128E-03	7.028694E-04	-6.057669E-05	3.709062E-06	-1.452709E-07	7.441239E-09
ROW 3	3.392902E-04	2.063568E-04	-1.132065E-05	7.300262E-07	-2.758578E-08	6.053150E-10
ROW 4	-2.916401E-05	-6.983055E-06	5.105382E-07	-2.979013E-08	1.888160E-09	-4.91674[illegible]E-11
ROW 5	2.126412E-06	7.460290E-07	-4.102885E-08	2.478693E-09	-1.073409E-10	2.351240E-12
ROW 6	-9.989183E-08	-3.000796E-08	2.233785E-09	-1.210261E-10	7.312503E-12	-1.793687E-13

MATRIX T(3),IMAGINARY

ROW 1	-2.711147E-02	-2.398436E-02	1.319671E-03	-1.090857E-04	4.188621E-06	-1.926674E-07
ROW 2	-1.746369E-02	2.789191E-04	-8.386725E-04	2.995392E-05	-3.778878E-06	1.257407E-07
ROW 3	3.251811E-04	-7.063376E-04	1.785304E-05	-1.429498E-05	3.662616E-07	-5.436146E-08
ROW 4	-1.252955E-04	2.531820E-05	-1.594091E-05	3.549493E-07	-1.866896E-07	4.057898E-09
ROW 5	5.781378E-06	-3.817400E-06	4.151471E-07	-1.894678E-07	3.316896E-09	-1.697122E-09
ROW 6	-3.886992E-07	1.479290E-07	-6.462707E-08	4.119105E-09	-1.747525E-09	2.556549E-11

MATRIX T(4),REAL

ROW 1	2.687334E-01	1.642962E-02	4.209066E-03	-1.119041E-04	1.479300E-05	-5.356594E-07
ROW 2	1.642962E-02	4.158105E-03	2.797572E-04	3.118641E-05	-6.312220E-08	8.854098E-08
ROW 3	4.209066E-03	2.797572E-04	7.048080E-05	-1.360638E-06	2.607351E-07	-7.799830E-09
ROW 4	-1.119041E-04	3.118641E-05	-1.360638E-06	5.111917E-07	-1.701316E-08	1.698728E-09
ROW 5	1.479300E-05	-6.312220E-08	2.607351E-07	-1.701316E-08	1.438474E-09	-7.022444E-11
ROW 6	-5.356594E-07	8.854098E-08	-7.799830E-09	1.698728E-09	-7.022444E-11	5.910193E-12

MATRIX T(4),IMAGINARY

ROW 1	-4.405457E-01	-2.318274E-02	-6.861128E-03	2.273124E-04	-2.512507E-05	1.010034E-06
ROW 2	-2.318274E-02	-5.493161E-02	-1.002859E-03	-6.518983E-04	1.454534E-05	-2.061880E-06
ROW 3	-6.861128E-03	-1.002859E-03	-1.958490E-03	-3.885159E-05	-1.795566E-05	2.285950E-07
ROW 4	2.273124E-04	-6.518983E-04	-3.885159E-05	-4.525781E-05	-7.673277E-07	-3.215002E-07
ROW 5	-2.512507E-05	1.454534E-05	-1.795566E-05	-7.673277E-07	-6.732419E-07	-1.094353E-08
ROW 6	1.010034E-06	-2.061880E-06	2.285950E-07	-3.215002E-07	-1.094353E-08	-7.184437E-09

********** ACCUMULATED SUMS FOR M = 1 **********

CLASS 1 ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	1.742513E+00	-1.257703E+00	-1.742513E+00	-1.150645E+00	-8.465618E-01	2.040651E+00	-1.436570E+02
4.00	1.754010E+00	-1.258897E+00	-1.754010E+00	-1.157164E+00	-8.488272E-01	2.059535E+00	-1.437384E+02
8.00	1.788356E+00	-1.262546E+00	-1.788356E+00	-1.176552E+00	-8.556506E-01	2.116412E+00	-1.439734E+02
12.00	1.845109E+00	-1.268831E+00	-1.845109E+00	-1.208306E+00	-8.671093E-01	2.211881E+00	-1.443359E+02
16.00	1.923524E+00	-1.278024E+00	-1.923524E+00	-1.251593E+00	-8.833187E-01	2.346737E+00	-1.447873E+02
20.00	2.022540E+00	-1.290434E+00	-2.022540E+00	-1.305265E+00	-9.044129E-01	2.521680E+00	-1.452821E+02
24.00	2.140766E+00	-1.306355E+00	-2.140766E+00	-1.367876E+00	-9.305194E-01	2.736951E+00	-1.457739E+02
28.00	2.276465E+00	-1.326001E+00	-2.276465E+00	-1.437712E+00	-9.617286E-01	2.991936E+00	-1.462202E+02
32.00	2.427546E+00	-1.349458E+00	-2.427546E+00	-1.512835E+00	-9.980606E-01	3.284796E+00	-1.465860E+02
36.00	2.591565E+00	-1.376637E+00	-2.591565E+00	-1.591143E+00	-1.039432E+00	3.612154E+00	-1.468450E+02
40.00	2.765728E+00	-1.407251E+00	-2.765728E+00	-1.670433E+00	-1.085621E+00	3.968921E+00	-1.469800E+02
44.00	2.946922E+00	-1.440806E+00	-2.946922E+00	-1.748492E+00	-1.136242E+00	4.348273E+00	-1.469825E+02
48.00	3.131745E+00	-1.476615E+00	-3.131745E+00	-1.823163E+00	-1.190717E+00	4.741806E+00	-1.468515E+02
52.00	3.316571E+00	-1.513825E+00	-3.316571E+00	-1.892539E+00	-1.248262E+00	5.139865E+00	-1.465924E+02
56.00	3.497613E+00	-1.551456E+00	-3.497613E+00	-1.954852E+00	-1.307879E+00	5.531995E+00	-1.462158E+02
60.00	3.671017E+00	-1.588453E+00	-3.671017E+00	-2.008745E+00	-1.368361E+00	5.907468E+00	-1.457372E+02
64.00	3.832959E+00	-1.623735E+00	-3.832959E+00	-2.053229E+00	-1.428303E+00	6.255801E+00	-1.451761E+02
68.00	3.979755E+00	-1.656246E+00	-3.979755E+00	-2.087729E+00	-1.486137E+00	6.567215E+00	-1.445551E+02
72.00	4.107976E+00	-1.685003E+00	-4.107976E+00	-2.112078E+00	-1.540167E+00	6.832986E+00	-1.438998E+02
76.00	4.214554E+00	-1.709130E+00	-4.214554E+00	-2.126491E+00	-1.588619E+00	7.045673E+00	-1.432380E+02
80.00	4.296893E+00	-1.727894E+00	-4.296893E+00	-2.131502E+00	-1.629710E+00	7.199254E+00	-1.425991E+02
84.00	4.352958E+00	-1.740727E+00	-4.352958E+00	-2.127892E+00	-1.661709E+00	7.289199E+00	-1.420131E+02
88.00	4.381347E+00	-1.747241E+00	-4.381347E+00	-2.116600E+00	-1.683015E+00	7.312537E+00	-1.415100E+02
92.00	4.381347E+00	-1.747241E+00	-4.381347E+00	-2.098647E+00	-1.692228E+00	7.267955E+00	-1.411193E+02
96.00	4.352958E+00	-1.740727E+00	-4.352958E+00	-2.075055E+00	-1.688217E+00	7.155928E+00	-1.408690E+02
100.00	4.296893E+00	-1.727894E+00	-4.296893E+00	-2.046796E+00	-1.670182E+00	6.978883E+00	-1.407856E+02
104.00	4.214554E+00	-1.709130E+00	-4.214554E+00	-2.014761E+00	-1.637702E+00	6.741328E+00	-1.408940E+02
108.00	4.107976E+00	-1.685003E+00	-4.107976E+00	-1.979740E+00	-1.590765E+00	6.449904E+00	-1.412173E+02
112.00	3.979755E+00	-1.656246E+00	-3.979755E+00	-1.942433E+00	-1.529789E+00	6.113299E+00	-1.417773E+02
116.00	3.832959E+00	-1.623735E+00	-3.832959E+00	-1.903465E+00	-1.455612E+00	5.741986E+00	-1.425942E+02
120.00	3.671017E+00	-1.588453E+00	-3.671017E+00	-1.863410E+00	-1.369480E+00	5.347772E+00	-1.436866E+02
124.00	3.497613E+00	-1.551456E+00	-3.497613E+00	-1.822812E+00	-1.273007E+00	4.943191E+00	-1.450705E+02
128.00	3.316571E+00	-1.513825E+00	-3.316571E+00	-1.782205E+00	-1.168128E+00	4.540779E+00	-1.467576E+02
132.00	3.131745E+00	-1.476615E+00	-3.131745E+00	-1.742122E+00	-1.057040E+00	4.152324E+00	-1.487525E+02
136.00	2.946922E+00	-1.440806E+00	-2.946922E+00	-1.703094E+00	-9.421367E-01	3.788152E+00	-1.510490E+02
140.00	2.765728E+00	-1.407251E+00	-2.765728E+00	-1.665645E+00	-8.259378E-01	3.456547E+00	-1.536247E+02
144.00	2.591565E+00	-1.376637E+00	-2.591565E+00	-1.630278E+00	-7.110211E-01	3.163356E+00	-1.564363E+02
148.00	2.427546E+00	-1.349458E+00	-2.427546E+00	-1.597461E+00	-5.999514E-01	2.911823E+00	-1.594155E+02
152.00	2.276465E+00	-1.326001E+00	-2.276465E+00	-1.567616E+00	-4.952156E-01	2.702660E+00	-1.624685E+02
156.00	2.140766E+00	-1.306355E+00	-2.140766E+00	-1.541108E+00	-3.991606E-01	2.534343E+00	-1.654790E+02
160.00	2.022540E+00	-1.290434E+00	-2.022540E+00	-1.518237E+00	-3.139383E-01	2.403602E+00	-1.683171E+02
164.00	1.923524E+00	-1.278024E+00	-1.923524E+00	-1.499243E+00	-2.414556E-01	2.306032E+00	-1.708510E+02
168.00	1.845109E+00	-1.268831E+00	-1.845109E+00	-1.484306E+00	-1.833321E-01	2.236776E+00	-1.729588E+02
172.00	1.788356E+00	-1.262546E+00	-1.788356E+00	-1.473555E+00	-1.408649E-01	2.191206E+00	-1.745394E+02
176.00	1.754010E+00	-1.258897E+00	-1.754010E+00	-1.467072E+00	-1.150003E-01	2.165526E+00	-1.755179E+02
180.00	1.742513E+00	-1.257703E+00	-1.742513E+00	-1.464906E+00	-1.063144E-01	2.157253E+00	-1.758491E+02

********** ACCUMULATED SUMS FOR M = 1 **********

CLASS 2 ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	1.742513E+00	1.257703E+00	1.742513E+00	-1.150645E+00	-8.465618E-01	2.040651E+00	-1.436570E+02
4.00	1.744480E+00	1.259202E+00	1.744480E+00	-1.154823E+00	-8.472257E-01	2.051407E+00	-1.437347E+02
8.00	1.750357E+00	1.263600E+00	1.750357E+00	-1.167297E+00	-8.491773E-01	2.083684E+00	-1.439651E+02
12.00	1.760074E+00	1.270606E+00	1.760074E+00	-1.187888E+00	-8.522997E-01	2.137492E+00	-1.443409E+02
16.00	1.773513E+00	1.279768E+00	1.773513E+00	-1.216294E+00	-8.564045E-01	2.212799E+00	-1.448502E+02
20.00	1.790504E+00	1.290509E+00	1.790504E+00	-1.252084E+00	-8.612430E-01	2.309453E+00	-1.454779E+02
24.00	1.810821E+00	1.302180E+00	1.810821E+00	-1.294695E+00	-8.665187E-01	2.427391E+00	-1.462062E+02
28.00	1.834181E+00	1.314119E+00	1.834181E+00	-1.343431E+00	-8.719026E-01	2.565021E+00	-1.470160E+02
32.00	1.860238E+00	1.325694E+00	1.860238E+00	-1.397460E+00	-8.770487E-01	2.722108E+00	-1.478875E+02
36.00	1.888584E+00	1.336357E+00	1.888584E+00	-1.455824E+00	-8.816095E-01	2.896658E+00	-1.488020E+02
40.00	1.918749E+00	1.345677E+00	1.918749E+00	-1.517455E+00	-8.852487E-01	3.086334E+00	-1.497417E+02
44.00	1.950200E+00	1.353360E+00	1.950200E+00	-1.581193E+00	-8.876520E-01	3.288098E+00	-1.506910E+02
48.00	1.982353E+00	1.359255E+00	1.982353E+00	-1.645820E+00	-8.885337E-01	3.498215E+00	-1.516366E+02
52.00	2.014577E+00	1.363357E+00	2.014577E+00	-1.710089E+00	-8.876398E-01	3.712308E+00	-1.525679E+02
56.00	2.046209E+00	1.365780E+00	2.046209E+00	-1.772767E+00	-8.847479E-01	3.925482E+00	-1.534773E+02
60.00	2.076566E+00	1.366745E+00	2.076568E+00	-1.832674E+00	-8.796631E-01	4.132500E+00	-1.543595E+02
64.00	2.104974E+00	1.366542E+00	2.104974E+00	-1.888713E+00	-8.722137E-01	4.327994E+00	-1.552124E+02
68.00	2.130767E+00	1.365505E+00	2.130767E+00	-1.939907E+00	-8.622463E-01	4.506709E+00	-1.560359E+02
72.00	2.153331E+00	1.363980E+00	2.153331E+00	-1.985415E+00	-8.496223E-01	4.663729E+00	-1.568324E+02
76.00	2.172109E+00	1.362300E+00	2.172109E+00	-2.024543E+00	-8.342173E-01	4.794691E+00	-1.576058E+02
80.00	2.186632E+00	1.360758E+00	2.186632E+00	-2.056751E+00	-8.159249E-01	4.895957E+00	-1.583615E+02
84.00	2.196527E+00	1.359592E+00	2.196527E+00	-2.081646E+00	-7.946645E-01	4.964740E+00	-1.591057E+02
88.00	2.201540E+00	1.358966E+00	2.201540E+00	-2.098975E+00	-7.703926E-01	4.999199E+00	-1.598452E+02
92.00	2.201540E+00	1.358966E+00	2.201540E+00	-2.108616E+00	-7.431170E-01	4.998483E+00	-1.605866E+02
96.00	2.196527E+00	1.359592E+00	2.196527E+00	-2.110572E+00	-7.129115E-01	4.962755E+00	-1.613360E+02
100.00	2.186632E+00	1.360758E+00	2.186632E+00	-2.104965E+00	-6.799290E-01	4.893183E+00	-1.620989E+02
104.00	2.172109E+00	1.362300E+00	2.172109E+00	-2.092043E+00	-6.444110E-01	4.791909E+00	-1.628796E+02
108.00	2.153331E+00	1.363980E+00	2.153331E+00	-2.072174E+00	-6.066913E-01	4.661980E+00	-1.636810E+02
112.00	2.130767E+00	1.365505E+00	2.130767E+00	-2.045859E+00	-5.671925E-01	4.507247E+00	-1.645045E+02
116.00	2.104974E+00	1.366542E+00	2.104974E+00	-2.013728E+00	-5.264160E-01	4.332212E+00	-1.653499E+02
120.00	2.076568E+00	1.366745E+00	2.076568E+00	-1.976537E+00	-4.849244E-01	4.141849E+00	-1.662153E+02
124.00	2.046209E+00	1.365780E+00	2.046209E+00	-1.935160E+00	-4.433197E-01	3.941375E+00	-1.670969E+02
128.00	2.014577E+00	1.363357E+00	2.014577E+00	-1.890568E+00	-4.022177E-01	3.736028E+00	-1.679894E+02
132.00	1.982353E+00	1.359255E+00	1.982353E+00	-1.843807E+00	-3.622220E-01	3.530829E+00	-1.688856E+02
136.00	1.950200E+00	1.353360E+00	1.950200E+00	-1.795962E+00	-3.238998E-01	3.330392E+00	-1.697767E+02
140.00	1.918749E+00	1.345677E+00	1.918749E+00	-1.748130E+00	-2.877611E-01	3.138764E+00	-1.706523E+02
144.00	1.888584E+00	1.336357E+00	1.888584E+00	-1.701380E+00	-2.542445E-01	2.959335E+00	-1.715009E+02
148.00	1.860238E+00	1.325694E+00	1.860238E+00	-1.656730E+00	-2.237080E-01	2.794800E+00	-1.723099E+02
152.00	1.834181E+00	1.314119E+00	1.834181E+00	-1.615116E+00	-1.964277E-01	2.647185E+00	-1.730658E+02
156.00	1.810821E+00	1.302180E+00	1.810821E+00	-1.577378E+00	-1.726013E-01	2.517911E+00	-1.737554E+02
160.00	1.790504E+00	1.290509E+00	1.790504E+00	-1.544241E+00	-1.523580E-01	2.407894E+00	-1.743653E+02
164.00	1.773513E+00	1.279768E+00	1.773513E+00	-1.516319E+00	-1.357711E-01	2.317656E+00	-1.748834E+02
168.00	1.760074E+00	1.270606E+00	1.760074E+00	-1.494102E+00	-1.228727E-01	2.247439E+00	-1.752987E+02
172.00	1.750357E+00	1.263600E+00	1.750357E+00	-1.477969E+00	-1.136685E-01	2.197314E+00	-1.756021E+02
176.00	1.744480E+00	1.259202E+00	1.744480E+00	-1.468185E+00	-1.081520E-01	2.167264E+00	-1.757870E+02
180.00	1.742513E+00	1.257703E+00	1.742513E+00	-1.464906E+00	-1.063144E-01	2.157253E+00	-1.758491E+02

N = 2

REAL PART OF Q1(NORMALIZED,TRANSPOSED)

ROW 1	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	-.000000E 00	-1.285931E-02	-5.718617E-04	-1.262179E-04	1.617176E-06	-3.632677E-07
ROW 3	.000000E 00	-5.718617E-04	-5.820695E-04	-2.121382E-05	-5.071398E-06	3.690005E-08
ROW 4	-.000000E 00	-1.262179E-04	-2.121382E-05	-1.471164E-05	-4.342217E-07	-1.048265E-07
ROW 5	.000000E 00	1.617176E-06	-5.071398E-06	-4.342217E-07	-2.374929E-07	-5.819457E-09
ROW 6	-.000000E 00	-3.632677E-07	3.690005E-08	-1.048265E-07	-5.819457E-09	-2.661320E-09

IMAGINARY PART OF Q1(NORMALIZED,TRANSPOSED)

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00
ROW 2	.000000E 00	6.006871E-01	1.538242E-01	1.169331E-01	-5.969557E+00	2.208317E+01
ROW 3	-.000000E 00	6.430992E-03	5.837652E-01	-5.533773E-02	4.113904E-01	-1.241397E+00
ROW 4	.000000E 00	5.295720E-04	6.487687E-04	5.721704E-01	-5.886486E-02	3.694955E-01
ROW 5	-.000000E 00	-1.950544E-05	2.957292E-04	-1.693886E-04	5.623191E-01	-6.928594E-02
ROW 6	.000000E 00	1.129103E-06	-1.121776E-05	1.972807E-04	-3.162321E-04	5.481905E-01

REAL PART OF Q2(NORMALIZED,TRANSPOSED)

ROW 1	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00
ROW 2	-.000000E 00	1.100635E-03	3.650192E-04	-6.016391E-06	1.650004E-06	-6.199754E-08
ROW 3	-.000000E 00	3.650192E-04	1.063367E-05	8.896078E-06	-1.517073E-07	3.633249E-08
ROW 4	.000000E 00	-6.016391E-06	8.896078E-06	2.358921E-08	1.301362E-07	-1.967903E-09
ROW 5	-.000000E 00	1.650004E-06	-1.517073E-07	1.301362E-07	-5.485412E-10	1.284772E-09
ROW 6	.000000E 00	-6.199754E-08	3.633249E-08	-1.967903E-09	1.284772E-09	-7.990583E-12

IMAGINARY PART OF Q2(NORMALIZED,TRANSPOSED)

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00
ROW 2	.000000E 00	-1.074392E-02	-4.998052E-03	-1.741525E-01	-2.407710E-01	1.700511E+01
ROW 3	.000000E 00	-4.998052E-03	8.377618E-04	-1.687382E-03	-1.045580E-01	-6.705992E-01
ROW 4	-.000000E 00	2.454955E-04	-1.687381E-03	7.143379E-04	-6.836783E-04	-7.379619E-02
ROW 5	.000000E 00	-1.597296E-05	7.478045E-05	-6.836761E-04	3.912718E-04	-3.098182E-04
ROW 6	-.000000E 00	6.608159E-07	-4.177438E-06	2.554005E-05	-3.098013E-04	2.027106E-04

REAL SECTION OF ORTHOGONALIZED $\hat{O}$ MATRIX.

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00				
ROW 2	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00
	-.000000E 00	.000000E 00	.000000E 00	.000000E 00				
ROW 3	-.000000E 00	-.000000E 00	-2.097676E-02	3.338536E-03	-7.114265E-04	7.374721E-04	-1.985356E-04	9.483118E-06
	2.581599E-06	3.206567E-06	-5.606921E-07	-8.547754E-08				
ROW 4	.000000E 00	.000000E 00	2.962049E-03	3.313809E-02	8.914883E-04	1.860129E-03	-1.622539E-05	3.431795E-04
	3.449926E-06	-4.220465E-06	-1.156636E-07	1.003780E-06				
ROW 5	.000000E 00	.000000E 00	-1.006496E-03	6.192523E-04	-9.942722E-04	1.465778E-05	-3.854137E-05	1.573707E-05
	-8.489968E-06	-2.570311E-07	5.004972E-08	7.008340E-08				
ROW 6	-.000000E 00	-.000000E 00	8.811823E-04	1.329203E-03	1.971720E-05	1.404471E-03	2.077742E-05	4.685798E-05
	-4.413805E-07	1.254823E-05	8.055880E-08	-1.141858E-07				
ROW 7	.000000E 00	.000000E 00	-2.198991E-04	-1.094838E-05	-3.810959E-05	1.535213E-05	-2.567669E-05	-5.806048E-08
	-7.994907E-07	2.342222E-07	-1.832621E-07	-3.886085E-09				
ROW 8	.000000E 00	.000000E 00	-1.481660E-05	2.963603E-04	2.090040E-05	4.793756E-05	-2.570178E-08	3.446961E-05
	2.949344E-07	9.469896E-07	-5.105872E-09	2.450754E-07				
ROW 9	-.000000E 00	-.000000E 00	2.787735E-06	2.927678E-06	-9.010354E-06	-2.805957E-07	-7.964169E-07	2.297199E-07
	-4.236344E-07	-1.740005E-09	-1.066407E-08	2.355608E-09				
ROW 10	-.000000E 00	-.000000E 00	3.791220E-06	-3.823287E-06	-3.593512E-07	1.160132E-05	2.977991E-07	9.553053E-07
	-1.720163E-09	5.406314E-07	2.840768E-09	1.281534E-08				
ROW 11	-.000000E 00	-.000000E 00	-6.625545E-07	-1.137529E-07	6.724663E-08	6.634396E-08	-1.912192E-07	-3.779707E-09
	-1.061808E-08	2.333119E-09	-4.854721E-09	-1.817840E-11				
ROW 12	.000000E 00	.000000E 00	-1.372204E-07	8.040278E-07	8.041545E-08	-8.167163E-08	-4.355599E-09	2.320147E-07
	2.843612E-09	1.288032E-08	-1.768572E-11	5.890355E-09				

IMAGINARY SECTION OF ORTHOGONALIZED $\hat{Q}$ MATRIX.

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	.000000E 00	-.000000E 00	.000000E 00				
ROW 2	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00
	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00				
ROW 3	-.000000E 00	-.000000E 00	9.991738E-01	3.033164E-02	-1.090012E-02	1.262994E-02	-8.871851E-04	-5.675971E-04
	3.840107E-05	4.470733E-05	-1.841031E-06	-2.007214E-06				
ROW 4	.000000E 00	.000000E 00	-3.046095E-02	9.988309E-01	8.797560E-03	1.478011E-02	-3.642855E-04	1.309389E-03
	2.204028E-05	-3.860126E-05	-7.831897E-07	2.863662E-06				
ROW 5	.000000E 00	.000000E 00	1.117753E-02	-8.446238E-03	9.998879E-01	-2.844654E-03	-1.219281E-03	3.960614E-03
	-5.182574E-04	-1.766440E-04	2.027663E-05	1.083508E-05				
ROW 6	-.000000E 00	-.000000E 00	-1.212771E-02	-1.523947E-02	2.849416E-03	9.997985E-01	2.940986E-03	1.395493E-03
	-1.323714E-04	6.833980E-04	6.916847E-06	-2.494658E-05				
ROW 7	.000000E 00	.000000E 00	9.192531E-04	4.290443E-04	1.212505E-03	-2.925380E-03	9.999911E-01	-1.996872E-03
	2.532732E-04	1.569346E-03	-3.587256E-04	-5.808005E-05				
ROW 8	.000000E 00	.000000E 00	5.806439E-04	-1.246624E-03	-3.979216E-03	-1.402219E-03	1.998003E-03	9.999872E-01
	1.213416E-03	-4.544841E-04	-4.711868E-05	4.376924E-04				
ROW 9	-.000000E 00	-.000000E 00	-3.437345E-05	-2.833453E-05	5.231229E-04	1.331886E-04	-2.540756E-04	-1.211398E-03
	9.999982E-01	-1.002019E-03	5.762042E-04	6.856185E-04				
ROW 10	-.000000E 00	-.000000E 00	-3.672833E-05	4.496935E-05	1.723176E-04	-6.797054E-04	-1.570718E-03	4.565542E-04
	1.002325E-03	9.999975E-01	5.665121E-04	-6.991818E-04				
ROW 11	-.000000E 00	-.000000E 00	2.058493E-0[illegible]	1.207495E-06	-2.045495E-05	-7.640732E-06	3.598300E-04	4.694711E-05
	-5.763041E-04	-5.657072E-04	9.999995E-[illegible]	-4.486626E-04				
ROW 12	-.000000E 00	-.000000E 00	1.462597E-06	-2.499066E-06	-9.246009E-06	2.482761E-05	5.652833E-05	-4.366455E-04
	-6.856896E-04	6.999229E-04	4.486635E-04	9.999993E-01				

MATRIX T(1),REAL

ROW 1	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	-.000000E 00	4.506364E-04	1.859021E-05	4.179320E-06	-3.548983E-08	1.147988E-08
ROW 3	.000000E 00	1.859021E-05	2.291816E-06	1.664941E-07	9.747696E-09	2.545759E-10
ROW 4	-.000000E 00	4.179320E-06	1.664941E-07	4.225683E-08	-2.302609E-10	1.176548E-10
ROW 5	.000000E 00	-3.548983E-08	9.747696E-09	-2.302609E-10	9.202441E-11	-2.162042E-12
ROW 6	-.000000E 00	1.147988E-08	2.545759E-10	1.176548E-10	-2.162042E-12	3.705048E-13

MATRIX T(1),IMAGINARY

ROW 1	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	-.000000E 00	2.107181E-02	7.493695E-04	1.985837E-04	-2.383067E-06	5.572944E-07
ROW 3	.000000E 00	7.493695E-04	9.786411E-04	3.648792E-05	8.490075E-06	-5.516083E-08
ROW 4	-.000000E 00	1.985837E-04	3.648792E-05	2.538670E-05	7.932958E-07	1.825588E-07
ROW 5	.000000E 00	-2.383067E-06	8.490075E-06	7.932958E-07	4.188913E-07	1.077169E-08
ROW 6	-.000000E 00	5.572944E-07	-5.516083E-08	1.825588E-07	1.077169E-08	4.790585E-09

MATRIX T(2),REAL

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	.000000E 00	.000000E 00
ROW 2	-.000000E 00	2.867099E-05	-8.741184E-06	8.425469E-07	-6.851214E-08	4.592408E-09
ROW 3	.000000E 00	2.658419E-05	1.147160E-06	2.851912E-07	-5.530068E-09	8.889804E-10
ROW 4	-.000000E 00	-1.196476E-06	-1.483712E-07	-7.083191E-09	-3.033892E-10	-4.294581E-12
ROW 5	.000000E 00	1.185190E-07	7.653183E-09	1.077978E-09	-9.715538E-12	2.812226E-12
ROW 6	-.000000E 00	-5.566235E-09	-5.177904E-10	-4.061309E-11	-3.579531E-13	-7.439304E-14

MATRIX T(2),IMAGINARY

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	.000000E 00	.000000E 00
ROW 2	-.000000E 00	-2.317323E-03	-6.633739E-04	1.350749E-06	-3.178168E-06	1.136766E-07
ROW 3	.000000E 00	-8.769226E-04	-2.681384E-05	-1.864765E-05	2.967211E-07	-7.853618E-08
ROW 4	-.000000E 00	2.223039E-05	-1.821041E-05	5.544575E-09	-2.711752E-07	4.127888E-09
ROW 5	.000000E 00	-3.641886E-06	3.310834E-07	-2.659008E-07	1.497578E-09	-2.658450E-09
ROW 6	-.000000E 00	1.342573E-07	-7.214912E-08	4.248415E-09	-2.594510E-09	1.706937E-11

MATRIX T(3),REAL

ROW 1	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	.000000E 00	2.867099E-05	2.658419E-05	-1.196476E-06	1.185190E-07	-5.566235E-09
ROW 3	-.000000E 00	-8.741184E-06	1.147160E-06	-1.483712E-07	7.653183E-09	-5.177904E-10
ROW 4	.000000E 00	8.425469E-07	2.851912E-07	-7.083191E-09	1.077978E-09	-4.061309E-11
ROW 5	.000000E 00	-6.851214E-08	-5.530068E-09	-3.033892E-10	-9.715538E-12	-3.579531E-13
ROW 6	.000000E 00	4.592408E-09	8.889804E-10	-4.294581E-12	2.812226E-12	-7.439304E-14

MATRIX T(3),IMAGINARY

ROW 1	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	.000000E 00	-2.317323E-03	-8.769226E-04	2.223039E-05	-3.641886E-06	1.342573E-07
ROW 3	-.000000E 00	-6.633739E-04	-2.681384E-05	-1.821041E-05	3.310834E-07	-7.214912E-08
ROW 4	.000000E 00	1.350749E-06	-1.864765E-05	5.544575E-09	-2.659008E-07	4.248415E-09
ROW 5	-.000000E 00	-3.178168E-06	2.967211E-07	-2.711752E-07	1.497578E-09	-2.594510E-09
ROW 6	-.000000E 00	1.136766E-07	-7.853618E-08	4.127888E-09	-2.658450E-09	1.706937E-11

MATRIX T(4),REAL

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	.000000E 00	.000000E 00
ROW 2	.000000E 00	1.111517E-03	6.599310E-05	1.148621E-05	-1.123569E-07	3.294223E-08
ROW 3	-.000000E 00	6.599310E-05	5.979368E-06	7.130555E-07	1.218927E-08	1.656615E-09
ROW 4	.000000E 00	1.148621E-05	7.130555E-07	1.214946E-07	-8.008811E-10	3.478804E-10
ROW 5	.000000E 00	-1.123569E-07	1.218927E-08	-8.008811E-10	1.868627E-10	-5.723177E-12
ROW 6	.000000E 00	3.294223E-08	1.656615E-09	3.478804E-10	-5.723177E-12	1.093112E-12

MATRIX T(4),IMAGINARY

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	.000000E 00	.000000E 00
ROW 2	.000000E 00	-3.317475E-02	-1.858743E-03	-3.421759E-04	4.308384E-06	-1.000855E-06
ROW 3	-.000000E 00	-1.858743E-03	-1.440833E-03	-5.194699E-05	-1.252217E-05	1.009463E-07
ROW 4	.000000E 00	-3.421759E-04	-5.194699E-05	-3.504103E-05	-9.558999E-07	-2.465616E-07
ROW 5	-.000000E 00	4.308384E-06	-1.252217E-05	-9.558999E-07	-5.495038E-07	-1.256662E-08
ROW 6	-.000000E 00	-1.000855E-06	1.009463E-07	-2.465616E-07	-1.256662E-08	-5.997161E-09

********** ACCUMULATED SUMS FOR M = 2 **********

CLASS 1 ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	1.742513E+00	-1.257703E+00	-1.742513E+00	-1.150645E+00	-8.465618E-01	2.040651E+00	-1.436570E+02
4.00	1.754077E+00	-1.259478E+00	-1.754077E+00	-1.156241E+00	-8.484881E-01	2.056826E+00	-1.437275E+02
8.00	1.788619E+00	-1.264744E+00	-1.788619E+00	-1.172886E+00	-8.543225E-01	2.105529E+00	-1.439307E+02
12.00	1.845689E+00	-1.273324E+00	-1.845689E+00	-1.200149E+00	-8.642254E-01	2.187244E+00	-1.442424E+02
16.00	1.924523E+00	-1.284932E+00	-1.924523E+00	-1.237323E+00	-8.784433E-01	2.302632E+00	-1.446270E+02
20.00	2.024039E+00	-1.299193E+00	-2.024039E+00	-1.283441E+00	-8.972775E-01	2.452327E+00	-1.450419E+02
24.00	2.142818E+00	-1.315659E+00	-2.142818E+00	-1.337294E+00	-9.210453E-01	2.636681E+00	-1.454433E+02
28.00	2.279096E+00	-1.333831E+00	-2.279096E+00	-1.397449E+00	-9.500325E-01	2.855481E+00	-1.457912E+02
32.00	2.430753E+00	-1.353187E+00	-2.430753E+00	-1.462378E+00	-9.844436E-01	3.107678E+00	-1.460523E+02
36.00	2.595317E+00	-1.373202E+00	-2.595317E+00	-1.530307E+00	-1.024352E+00	3.391137E+00	-1.462026E+02
40.00	2.769970E+00	-1.393370E+00	-2.769970E+00	-1.599469E+00	-1.069651E+00	3.702454E+00	-1.462272E+02
44.00	2.951578E+00	-1.413223E+00	-2.951578E+00	-1.668056E+00	-1.120017E+00	4.036850E+00	-1.461206E+02
48.00	3.136727E+00	-1.432344E+00	-3.136727E+00	-1.734301E+00	-1.174883E+00	4.388149E+00	-1.458848E+02
52.00	3.321782E+00	-1.450378E+00	-3.321782E+00	-1.796536E+00	-1.233416E+00	4.748856E+00	-1.455284E+02
56.00	3.502957E+00	-1.467036E+00	-3.502957E+00	-1.853251E+00	-1.294525E+00	5.110336E+00	-1.450651E+02
60.00	3.676407E+00	-1.482097E+00	-3.676407E+00	-1.903146E+00	-1.356874E+00	5.463071E+00	-1.445125E+02
64.00	3.838321E+00	-1.495404E+00	-3.838321E+00	-1.945169E+00	-1.418914E+00	5.796998E+00	-1.438909E+02
68.00	3.985033E+00	-1.506856E+00	-3.985033E+00	-1.978555E+00	-1.478928E+00	6.101906E+00	-1.432227E+02
72.00	4.113136E+00	-1.516399E+00	-4.113136E+00	-2.002842E+00	-1.535091E+00	6.367879E+00	-1.425314E+02
76.00	4.219584E+00	-1.524013E+00	-4.219584E+00	-2.017875E+00	-1.585537E+00	6.585747E+00	-1.418417E+02
80.00	4.301804E+00	-1.529703E+00	-4.301804E+00	-2.023803E+00	-1.628425E+00	6.747546E+00	-1.411786E+02
84.00	4.357778E+00	-1.533482E+00	-4.357778E+00	-2.021050E+00	-1.662016E+00	6.846940E+00	-1.405677E+02
88.00	4.386118E+00	-1.535367E+00	-4.386118E+00	-2.010285E+00	-1.684738E+00	6.879587E+00	-1.400350E+02
92.00	4.386118E+00	-1.535367E+00	-4.386118E+00	-1.992377E+00	-1.695249E+00	6.843435E+00	-1.396066E+02
96.00	4.357778E+00	-1.533482E+00	-4.357778E+00	-1.968344E+00	-1.692491E+00	6.738906E+00	-1.393092E+02
100.00	4.301804E+00	-1.529703E+00	-4.301804E+00	-1.939299E+00	-1.675734E+00	6.568967E+00	-1.391699E+02
104.00	4.219584E+00	-1.524013E+00	-4.219584E+00	-1.906394E+00	-1.644607E+00	6.339070E+00	-1.392163E+02
108.00	4.113136E+00	-1.516399E+00	-4.113136E+00	-1.870770E+00	-1.599117E+00	6.056957E+00	-1.394765E+02
112.00	3.985033E+00	-1.506856E+00	-3.985033E+00	-1.833513E+00	-1.539662E+00	5.732328E+00	-1.399787E+02
116.00	3.838321E+00	-1.495404E+00	-3.838321E+00	-1.795616E+00	-1.467023E+00	5.376392E+00	-1.407511E+02
120.00	3.676407E+00	-1.482097E+00	-3.676407E+00	-1.757954E+00	-1.382355E+00	5.001306E+00	-1.418205E+02
124.00	3.502957E+00	-1.467036E+00	-3.502957E+00	-1.721270E+00	-1.287158E+00	4.619548E+00	-1.432110E+02
128.00	3.321782E+00	-1.450378E+00	-3.321782E+00	-1.686169E+00	-1.183251E+00	4.243247E+00	-1.449411E+02
132.00	3.136727E+00	-1.432344E+00	-3.136727E+00	-1.653116E+00	-1.072722E+00	3.883527E+00	-1.470201E+02
136.00	2.951578E+00	-1.413223E+00	-2.951578E+00	-1.622456E+00	-9.578838E-01	3.549906E+00	-1.494428E+02
140.00	2.769970E+00	-1.393370E+00	-2.769970E+00	-1.594420E+00	-8.412139E-01	3.249816E+00	-1.521839E+02
144.00	2.595317E+00	-1.373202E+00	-2.595317E+00	-1.569146E+00	-7.252935E-01	2.988269E+00	-1.551926E+02
148.00	2.430753E+00	-1.353187E+00	-2.430753E+00	-1.546697E+00	-6.127401E-01	2.767724E+00	-1.583885E+02
152.00	2.279096E+00	-1.333831E+00	-2.279096E+00	-1.527084E+00	-5.061388E-01	2.588161E+00	-1.616627E+02
156.00	2.142818E+00	-1.315659E+00	-2.142818E+00	-1.510273E+00	-4.079728E-01	2.447366E+00	-1.648834E+02
160.00	2.024039E+00	-1.299193E+00	-2.024039E+00	-1.496210E+00	-3.205557E-01	2.341401E+00	-1.679075E+02
164.00	1.924523E+00	-1.284932E+00	-1.924523E+00	-1.484829E+00	-2.459676E-01	2.265217E+00	-1.705941E+02
168.00	1.845689E+00	-1.273324E+00	-1.845689E+00	-1.476062E+00	-1.859972E-01	2.213353E+00	-1.728181E+02
172.00	1.788619E+00	-1.264744E+00	-1.788619E+00	-1.469847E+00	-1.420912E-01	2.180641E+00	-1.744783E+02
176.00	1.754077E+00	-1.259478E+00	-1.754077E+00	-1.466139E+00	-1.153133E-01	2.162861E+00	-1.755029E+02
180.00	1.742513E+00	-1.257703E+00	-1.742513E+00	-1.464906E+00	-1.063144E-01	2.157253E+00	-1.758491E+02

********** ACCUMULATED SUMS FOR M = 2 **********

CLASS 2 ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	1.742513E+00	1.257703E+00	1.742513E+00	-1.150645E+00	-8.465618E-01	2.040651E+00	-1.436570E+02
4.00	1.744547E+00	1.259799E+00	1.744547E+00	-1.153899E+00	-8.468856E-01	2.048699E+00	-1.437238E+02
8.00	1.750623E+00	1.266053E+00	1.750623E+00	-1.163617E+00	-8.478359E-01	2.072830E+00	-1.439221E+02
12.00	1.760666E+00	1.276362E+00	1.760666E+00	-1.179659E+00	-8.493504E-01	2.112992E+00	-1.442462E+02
16.00	1.774549E+00	1.290552E+00	1.774549E+00	-1.201797E+00	-8.513292E-01	2.169077E+00	-1.446870E+02
20.00	1.792090E+00	1.308380E+00	1.792090E+00	-1.229708E+00	-8.536403E-01	2.240882E+00	-1.452323E+02
24.00	1.813049E+00	1.329532E+00	1.813049E+00	-1.262979E+00	-8.561258E-01	2.328067E+00	-1.458681E+02
28.00	1.837126E+00	1.353627E+00	1.837126E+00	-1.301109E+00	-8.586095E-01	2.430095E+00	-1.465789E+02
32.00	1.863958E+00	1.380220E+00	1.863958E+00	-1.343508E+00	-8.609030E-01	2.546168E+00	-1.473488E+02
36.00	1.893118E+00	1.408808E+00	1.893118E+00	-1.389505E+00	-8.628120E-01	2.675167E+00	-1.481618E+02
40.00	1.924115E+00	1.438840E+00	1.924115E+00	-1.438349E+00	-8.641395E-01	2.815585E+00	-1.490032E+02
44.00	1.956400E+00	1.469727E+00	1.956400E+00	-1.489225E+00	-8.646890E-01	2.965477E+00	-1.498592E+02
48.00	1.989371E+00	1.500855E+00	1.989371E+00	-1.541256E+00	-8.642643E-01	3.122423E+00	-1.507183E+02
52.00	2.022380E+00	1.531601E+00	2.022380E+00	-1.593525E+00	-8.626691E-01	3.283521E+00	-1.515707E+02
56.00	2.054751E+00	1.561348E+00	2.054751E+00	-1.645087E+00	-8.597054E-01	3.445405E+00	-1.524089E+02
60.00	2.085789E+00	1.589496E+00	2.085789E+00	-1.694991E+00	-8.551722E-01	3.604313E+00	-1.532277E+02
64.00	2.114806E+00	1.615483E+00	2.114806E+00	-1.742300E+00	-8.488654E-01	3.756183E+00	-1.540242E+02
68.00	2.141133E+00	1.638789E+00	2.141133E+00	-1.786122E+00	-8.405790E-01	3.8968[illegible]E+00	-1.547975E+02
72.00	2.164148E+00	1.658951E+00	2.164148E+00	-1.825627E+00	-8.301090E-01	4.021996E+00	-1.555488E+02
76.00	2.183292E+00	1.675574E+00	2.183292E+00	-1.860082E+00	-8.172595E-01	4.127818E+00	-1.562809E+02
80.00	2.198090E+00	1.688332E+00	2.198090E+00	-1.888867E+00	-8.018520E-01	4.210786E+00	-1.569981E+02
84.00	2.208170E+00	1.696976E+00	2.208170E+00	-1.911503E+00	-7.837357E-01	4.268084E+00	-1.577059E+02
88.00	2.213276E+00	1.701340E+00	2.213276E+00	-1.927663E+00	-7.627998E-01	4.297748E+00	-1.584107E+02
92.00	2.213276E+00	1.701340E+00	2.213276E+00	-1.937187E+00	-7.389857E-01	4.298792E+00	-1.591195E+02
96.00	2.208170E+00	1.696976E+00	2.208170E+00	-1.940082E+00	-7.122984E-01	4.271287E+00	-1.598394E+02
100.00	2.198090E+00	1.688332E+00	2.198090E+00	-1.936521E+00	-6.828167E-01	4.216352E+00	-1.605774E+02
104.00	2.183292E+00	1.675574E+00	2.183292E+00	-1.926831E+00	-6.506993E-01	4.136087E+00	-1.613399E+02
108.00	2.164148E+00	1.658951E+00	2.164148E+00	-1.911477E+00	-6.161882E-01	4.033434E+00	-1.621326E+02
112.00	2.141133E+00	1.638789E+00	2.141133E+00	-1.891043E+00	-5.796071E-01	3.911987E+00	-1.629596E+02
116.00	2.114806E+00	1.615483E+00	2.114806E+00	-1.866202E+00	-5.413565E-01	3.775778E+00	-1.638234E+02
120.00	2.085789E+00	1.589496E+00	2.085789E+00	-1.837701E+00	-5.019026E-01	3.629051E+00	-1.647242E+02
124.00	2.054751E+00	1.561348E+00	2.054751E+00	-1.806326E+00	-4.617648E-01	3.476041E+00	-1.656602E+02
128.00	2.022380E+00	1.531601E+00	2.022380E+00	-1.772986E+00	-4.214982E-01	3.320787E+00	-1.666264E+02
132.00	1.989371E+00	1.500855E+00	1.989371E+00	-1.738191E+00	-3.816754E-01	3.165985E+00	-1.676154E+02
136.00	1.956400E+00	1.469727E+00	1.956400E+00	-1.703033E+00	-3.428680E-01	3.017879E+00	-1.686169E+02
140.00	1.924115E+00	1.438840E+00	1.924115E+00	-1.668173E+00	-3.056286E-01	2.876209E+00	-1.696179E+02
144.00	1.893118E+00	1.408808E+00	1.893118E+00	-1.634330E+00	-2.704746E-01	2.744192E+00	-1.706030E+02
148.00	1.863958E+00	1.380220E+00	1.863958E+00	-1.602173E+00	-2.378764E-01	2.623544E+00	-1.715549E+02
152.00	1.837126E+00	1.353627E+00	1.837126E+00	-1.572313E+00	-2.082477E-01	2.515534E+00	-1.724553E+02
156.00	1.813049E+00	1.329532E+00	1.813049E+00	-1.545296E+00	-1.819416E-01	2.421043E+00	-1.732850E+02
160.00	1.792090E+00	1.308380E+00	1.792090E+00	-1.521606E+00	-1.592492E-01	2.340644E+00	-1.740253E+02
164.00	1.774549E+00	1.290552E+00	1.774549E+00	-1.501653E+00	-1.404020E-01	2.274673E+00	-1.746585E+02
168.00	1.760666E+00	1.276362E+00	1.760666E+00	-1.485777E+00	-1.255770E-01	2.223304E+00	-1.751689E+02
172.00	1.750623E+00	1.266053E+00	1.750623E+00	-1.474246E+00	-1.149028E-01	2.186605E+00	-1.755434E+02
176.00	1.744547E+00	1.259799E+00	1.744547E+00	-1.467251E+00	-1.084655E-01	2.164590E+00	-1.757721E+02
180.00	1.742513E+00	1.257703E+00	1.742513E+00	-1.464906E+00	-1.063144E-01	2.157253E+00	-1.758491E+02

N = 3

REAL PART OF Q1(NORMALIZED,TRANSPOSED)

ROW 1	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00
ROW 2	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	.000000E 00
ROW 3	-.000000E 00	-.000000E 00	-3.633017E-04	-1.916494E-05	-2.598992E-06	-5.793605E-09
ROW 4	.000000E 00	.000000E 00	-1.916494E-05	-9.964365E-06	-4.436426E-07	-6.590991E-08
ROW 5	-.000000E 00	-.000000E 00	-2.598992E-06	-4.436426E-07	-1.700061E-07	-6.173203E-09
ROW 6	.000000E 00	.000000E 00	-5.793605E-09	-6.590991E-08	-6.173203E-09	-1.991914E-09

IMAGINARY PART OF Q1(NORMALIZED,TRANSPOSED)

ROW 1	.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 3	.000000E 00	.000000E 00	5.515185E-01	1.784695E-01	5.820851E-02	-9.612511E+00
ROW 4	-.000000E 00	-.000000E 00	3.514700E-03	5.524111E-01	1.021643E-03	4.181253E-01
ROW 5	.000000E 00	.000000E 00	1.210927E-04	9.141077E-04	5.504908E-01	-3.343383E-02
ROW 6	-.000000E 00	-.000000E 00	-3.800205E-06	9.529041E-05	1.932907E-04	5.496236E-01

REAL PART OF Q2(NORMALIZED,TRANSPOSED)

ROW 1	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 3	-.000000E 00	-.000000E 00	3.026242E-05	6.088527E-06	1.712612E-08	2.118473E-08
ROW 4	-.000000E 00	-.000000E 00	6.088527E-06	3.590134E-07	1.084826E-07	-2.537229E-10
ROW 5	.000000E 00	.000000E 00	1.712612E-08	1.084826E-07	2.634640E-09	1.228086E-09
ROW 6	-.000000E 00	-.000000E 00	2.118473E-08	-2.537229E-10	1.228086E-09	1.319774E-11

IMAGINARY PART OF Q2(NORMALIZED,TRANSPOSED)

ROW 1	.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 2	.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
ROW 3	.000000E 00	.000000E 00	-5.739599E-03	-1.660457E-03	-1.775983E-01	-1.016863E-01
ROW 4	.000000E 00	.000000E 00	-1.660457E-03	-1.134538E-04	-8.470718E-04	-1.338663E-01
ROW 5	-.000000E 00	-.000000E 00	6.919463E-05	-8.470713E-04	3.719961E-04	-4.494914E-04
ROW 6	.000000E 00	.000000E 00	-3.352165E-06	3.315171E-05	-4.494992E-04	3.182203E-04

REAL SECTION OF ORTHOGONALIZED $\hat{Q}$ MATRIX.

ROW 1	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00				
ROW 2	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00				
ROW 3	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00				
ROW 4	.000000E 00	.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00
	-.000000E 00	-.000000E 00	.000000E 00	.000000E 00				
ROW 5	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-6.466648E-04	7.280056E-05	-3.005755E-05	1.247625E-05
	-4.512615E-06	2.097664E-07	-7.075780E-09	4.278923E-08				
ROW 6	.000000E 00	.000000E 00	.000000E 00	.000000E 00	7.013051E-05	8.304983E-04	1.349773E-05	4.962779E-05
	2.907163E-08	6.126981E-06	4.352059E-08	1.899397E-08				
ROW 7	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-3.465681E-05	1.103233E-05	-1.794554E-05	6.822147E-07
	-8.056095E-07	2.014488E-07	-1.167285E-07	2.171685E-10				
ROW 8	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	1.362595E-05	4.287704E-05	7.636863E-07	2.240807E-05
	2.382187E-07	9.871467E-07	-1.213510E-09	1.511818E-07				
ROW 9	.000000E 00	.000000E 00	.000000E 00	.000000E 00	-4.721731E-06	2.462896E-08	-8.134972E-07	1.958014E-07
	-3.095011E-07	4.834286E-09	-1.122165E-08	2.272625E-09				
ROW 10	.000000E 00	.000000E 00	.000000E 00	.000000E 00	3.460021E-08	5.780764E-06	2.411446E-07	9.760917E-07
	5.637961E-09	3.771810E-07	2.670152E-09	1.372652E-08				
ROW 11	.000000E 00	.000000E 00	.000000E 00	.000000E 00	-1.060151E-08	3.852753E-08	-1.199176E-07	-6.497632E-10
	-1.123521E-08	2.216790E-09	-3.624175E-09	2.488919E-11				
ROW 12	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	4.703778E-08	1.286391E-08	-5.633570E-10	1.463439E-07
	2.726798E-09	1.370675E-08	2.930378E-11	4.422771E-09				

IMAGINARY SECTION OF ORTHOGONALIZED $\hat{Q}$ MATRIX.

ROW 1	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	.000000E 00	.000000E 00	.000000E 00				
ROW 2	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	.000000E 00	.000000E 00	.000000E 00				
ROW 3	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	.000000E 00	-.000000E 00	.000000E 00
	.000000E 00	.000000E 00	.000000E 00	.000000E 00				
ROW 4	.000000E 00	.000000E 00	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	.000000E 00	-.000000E 00
	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00				
ROW 5	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	9.998742E-01	1.401803E-02	-6.322523E-03	3.827164E-03
	-2.047364E-04	-1.547119E-04	7.804781E-06	8.322731E-06				
ROW 6	.000000E 00	.000000E 00	.000000E 00	.000000E 00	-1.402821E-02	9.998660E-01	3.091226E-03	7.795559E-03
	-1.156146E-04	2.938150E-04	5.280102E-06	-7.112329E-06				
ROW 7	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	6.364673E-03	-3.004058E-03	9.999720E-01	1.340878E-04
	-1.673036E-03	1.889602E-03	-1.711748E-04	-7.424360E-05				
ROW 8	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-3.717679E-03	-7.848475E-03	-1.352327E-04	9.999590E-01
	1.535930E-03	2.014436E-03	-5.938139E-05	2.140588E-04				
ROW 9	.000000E 00	.000000E 00	.000000E 00	.000000E 00	2.193007E-04	1.257528E-04	1.673982E-03	-1.532375E-03
	9.999964E-01	-9.020622E-04	-3.531878E-04	9.974642E-04				
ROW 10	.000000E 00	.000000E 00	.000000E 00	.000000E 00	1.544658E-04	-2.700146E-04	-1.889489E-03	-2.017739E-03
	9.019930E-04	9.999953E-01	8.177101E-04	4.292120E-04				
ROW 11	.000000E 00	.000000E 00	.000000E 00	.000000E 00	-6.904629E-06	-6.109867E-06	1.732794E-04	6.058912E-05
	3.529614E-04	-8.172794E-04	9.999993E-01	-7.065631E-04				
ROW 12	.000000E 00	.000000E 00	.000000E 00	.000000E 00	-7.443022E-06	8.437835E-06	7.360883E-05	-2.115793E-04
	-9.980514E-04	-4.291755E-04	7.065645E-04	9.999991E-01				

MATRIX T(1),REAL

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00
ROW 2	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00
ROW 3	-.000000E 00	-.000000E 00	4.245027E-07	2.101995E-08	2.952816E-09	1.170980E-11
ROW 4	.000000E 00	.000000E 00	2.101995E-08	1.409005E-09	1.509241E-10	2.904146E-12
ROW 5	-.000000E 00	-.000000E 00	2.952816E-09	1.509241E-10	2.116625E-11	1.304729E-13
ROW 6	-.000000E 00	-.000000E 00	1.170980E-11	2.904146E-12	1.304729E-13	1.571731E-14

MATRIX T(1),IMAGINARY

ROW 1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00
ROW 2	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00
ROW 3	-.000000E 00	-.000000E 00	6.478395E-04	3.036031E-05	4.518535E-06	8.425856E-09
ROW 4	.000000E 00	.000000E 00	3.036031E-05	1.771521E-05	7.775287E-07	1.165702E-07
ROW 5	-.000000E 00	-.000000E 00	4.518535E-06	7.775287E-07	3.068674E-07	1.103066E-08
ROW 6	.000000E 00	.000000E 00	8.425856E-09	1.165702E-07	1.103066E-08	3.597778E-09

MATRIX T(2),REAL

ROW 1	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW 2	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW 3	-.000000E 00	-.000000E 00	1.136769E-08	-4.306722E-09	3.004999E-10	-2.429581E-11
ROW 4	.000000E 00	.000000E 00	8.857768E-09	2.998033E-10	7.362078E-11	-9.167498E-13
ROW 5	-.000000E 00	-.000000E 00	-3.030261E-10	-5.012401E-11	-6.949633E-13	-1.573140E-13
ROW 6	-.000000E 00	-.000000E 00	3.430386E-11	1.965141E-12	2.413981E-13	3.262398E-16

MATRIX T(2),IMAGINARY

ROW 1	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW 2	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW 3	-.000000E 00	-.000000E 00	-6.105272E-05	-1.169972E-05	-1.214609E-07	-4.195932E-08
ROW 4	.000000E 00	.000000E 00	-1.312232E-05	-7.521888E-07	-2.182201E-07	3.690547E-11
ROW 5	-.000000E 00	-.000000E 00	3.368017E-08	-2.215186E-07	-5.589428E-09	-2.501477E-09
ROW 6	.000000E 00	.000000E 00	-4.377364E-08	9.053424E-10	-2.474078E-09	-2.985549E-11

MATRIX T(3),REAL

ROW	1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00
ROW	2	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000E 00
ROW	3	-.000000E 00	-.000000E 00	1.136769E-08	8.857768E-09	-3.030261E-10	3.430386E-11
ROW	4	-.000000E 00	-.000000E 00	-4.306722E-09	2.998033E-10	-5.012401E-11	1.965141E-12
ROW	5	-.000000E 00	-.000000E 00	3.004999E-10	7.362078E-11	-6.949633E-13	2.413981F-13
ROW	6	-.000000E 00	-.000000E 00	-2.429581E-11	-9.167498E-13	-1.573140E-13	3.262398F-16

MATRIX T(3),IMAGINARY

ROW	1	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000F 00
ROW	2	.000000E 00	.000000E 00	-.000000E 00	.000000E 00	-.000000E 00	-.000000F 00
ROW	3	.000000E 00	.000000E 00	-6.105272E-05	-1.312232E-05	3.368017E-08	-4.377364E-08
ROW	4	-.000000E 00	-.000000E 00	-1.169972E-05	-7.521888E-07	-2.215186E-07	9.053424F-10
ROW	5	-.000000E 00	-.000000E 00	-1.214609E-07	-2.182201E-07	-5.589428E-09	-2.474078E-09
ROW	6	.000000E 00	.000000E 00	-4.195932E-08	3.690547E-11	-2.501477E-09	-2.985549E-11

MATRIX T(4),REAL

ROW	1	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW	2	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW	3	-.000000E 00	-.000000E 00	6.970210E-07	4.309804E-08	5.150448F-09	2.545363F-11
ROW	4	-.000000E 00	-.000000E 00	4.309804E-08	3.122174E-09	3.293142F-10	4.878810F-12
ROW	5	-.000000E 00	-.000000E 00	5.150448E-09	3.293142E-10	3.874142E-11	2.798829E-13
ROW	6	-.000000E 00	-.000000E 00	2.545363E-11	4.878810E-12	2.798829F-13	2.526083F-14

MATRIX T(4),IMAGINARY

ROW	1	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW	2	.000000E 00	.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00	-.000000E 00
ROW	3	.000000E 00	.000000E 00	-8.310364E-04	-4.961788E-05	-6.120647E-06	-1.840066E-08
ROW	4	-.000000E 00	-.000000E 00	-4.961788E-05	-2.283957E-05	-1.034928E-06	-1.514554E-07
ROW	5	-.000000E 00	-.000000E 00	-6.120647E-06	-1.034928E-06	-3.813042E-07	-1.402641E-08
ROW	6	.000000E 00	.000000E 00	-1.840066E-08	-1.514554E-07	-1.402641E-08	-4.463475E-09

********** ACCUMULATED SUMS FOR N = 3 **********

CLASS 1 ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	1.742513E+00	-1.257703E+00	-1.742513E+00	-1.150645E+00	-8.445618E-01	2.040651E+00	-1.436570E+02
4.00	1.754077E+00	-1.259478E+00	-1.754077E+00	-1.156241E+00	-8.484881E-01	2.05[illegible]26E+00	-1.437275E+02
8.00	1.788619E+00	-1.264745E+00	-1.788619E+00	-1.172889E+00	-8.543235E-01	2.105537E+00	-1.439707E+02
12.00	1.845689E+00	-1.273328E+00	-1.845689E+00	-1.200162E+00	-8.642307E-01	2.187284E+00	-1.442425E+02
16.00	1.924523E+00	-1.284941E+00	-1.924523E+00	-1.237364E+00	-8.784591E-01	2.302760E+00	-1.446276E+02
20.00	2.024039E+00	-1.299207E+00	-2.024039E+00	-1.283536E+00	-8.973134E-01	2.452636E+00	-1.450428E+02
24.00	2.142818E+00	-1.315670E+00	-2.142818E+00	-1.337483E+00	-9.211131E-01	2.637310E+00	1.454451E+02
28.00	2.279097E+00	-1.333819E+00	-2.279097E+00	-1.397799E+00	-9.501452E-01	2.856617E+00	-1.457943E+02
32.00	2.430754E+00	-1.353112E+00	-2.430754E+00	-1.462904E+00	-9.846131E-01	3.109550E+00	-1.460573E+02
36.00	2.595318E+00	-1.372998E+00	-2.595318E+00	-1.531087E+00	-1.024586E+00	3.394005E+00	-1.462100E+02
40.00	2.769972E+00	-1.392945E+00	-2.769972E+00	-1.600558E+00	-1.069953E+00	3.706585E+00	-1.462378E+02
44.00	2.951580E+00	-1.412460E+00	-2.951580E+00	-1.669499E+00	-1.120386E+00	4.042492E+00	-1.461348E+02
48.00	3.136729E+00	-1.431110E+00	-3.136729E+00	-1.736131E+00	-1.175306E+00	4.395498E+00	-1.459032E+02
52.00	3.321785E+00	-1.448530E+00	-3.321785E+00	-1.798770E+00	-1.233880E+00	4.758031E+00	-1.455515E+02
56.00	3.502961E+00	-1.464440E+00	-3.502961E+00	-1.855885E+00	-1.295009E+00	5.121356E+00	-1.450932E+02
60.00	3.676412E+00	-1.478639E+00	-3.676412E+00	-1.906156E+00	-1.357356E+00	5.475847E+00	-1.445457E+02
64.00	3.838326E+00	-1.491007E+00	-3.838326E+00	-1.948519E+00	-1.419372E+00	5.811343E+00	-1.439291E+02
68.00	3.985038E+00	-1.501492E+00	-3.985038E+00	-1.982197E+00	-1.479342E+00	6.117556E+00	-1.432655E+02
72.00	4.113141E+00	-1.510098E+00	-4.113141E+00	-2.006720E+00	-1.535447E+00	6.384519E+00	-1.425785E+02
76.00	4.219590E+00	-1.516867E+00	-4.219590E+00	-2.021934E+00	-1.585822E+00	6.603047E+00	-1.418926E+02
80.00	4.301810E+00	-1.521862E+00	-4.301810E+00	-2.027990E+00	-1.628632E+00	6.765186E+00	-1.412329E+02
84.00	4.357783E+00	-1.525146E+00	-4.357783E+00	-2.025318E+00	-1.662140E+00	6.864625E+00	-1.406249E+02
88.00	4.386124E+00	-1.526774E+00	-4.386124E+00	-2.014592E+00	-1.684778E+00	6.897059E+00	-1.400947E+02
92.00	4.386124E+00	-1.526774E+00	-4.386124E+00	-1.996684E+00	-1.695204E+00	6.860464E+00	-1.396684E+02
96.00	4.357783E+00	-1.525146E+00	-4.357783E+00	-1.972613E+00	-1.692362E+00	6.755288E+00	-1.393727E+02
100.00	4.301810E+00	-1.521862E+00	-4.301810E+00	-1.943486E+00	-1.675522E+00	6.584514E+00	-1.392346E+02
104.00	4.219590E+00	-1.516867E+00	-4.219590E+00	-1.910453E+00	-1.644317E+00	6.353608E+00	-1.392816E+02
108.00	4.113141E+00	-1.510098E+00	-4.113141E+00	-1.874648E+00	-1.598758E+00	6.070330E+00	-1.395414E+02
112.00	3.985038E+00	-1.501492E+00	-3.985038E+00	-1.837154E+00	-1.539244E+00	5.744408E+00	-1.400423E+02
116.00	3.838326E+00	-1.491007E+00	-3.838326E+00	-1.798966E+00	-1.466563E+00	5.387085E+00	-1.408122E+02
120.00	3.676412E+00	-1.478639E+00	-3.676412E+00	-1.760964E+00	-1.381871E+00	5.0[illegible]0564E+00	-1.418779E+02
124.00	3.502961E+00	-1.464440E+00	-3.502961E+00	-1.723903E+00	-1.286673E+00	4.627372E+00	-1.432634E+02
128.00	3.321785E+00	-1.448530E+00	-3.321785E+00	-1.688402E+00	-1.182787E+00	4.249686E+00	-1.449874E+02
132.00	3.136729E+00	-1.431110E+00	-3.136729E+00	-1.654947E+00	-1.072298E+00	3.888673E+00	-1.470594E+02
136.00	2.951580E+00	-1.412460E+00	-2.951580E+00	-1.623900E+00	-9.575154E-01	3.553886E+00	-1.494747E+02
140.00	2.769972E+00	-1.392945E+00	-2.769972E+00	-1.595508E+00	-8.409109E-01	3.252778E+00	-1.522086E+02
144.00	2.595318E+00	-1.372998E+00	-2.595318E+00	-1.569926E+00	-7.250590E-01	2.990377E+00	[illegible]2105E+02
148.00	2.430754E+00	-1.353112E+00	-2.430754E+00	-1.547224E+00	-6.125707E-01	2.769144E+00	-1.584006E+02
152.00	2.279097E+00	-1.333819E+00	-2.279097E+00	-1.527413E+00	-5.060261E-01	2.589054E+00	-1.616702E+02
156.00	2.142818E+00	-1.315670E+00	-2.142818E+00	-1.510461E+00	-4.079050E-01	2.447880E+00	-1.648876E+02
160.00	2.024039E+00	-1.299207E+00	-2.024039E+00	-1.496305E+00	-3.205199E-01	2.341663E+00	-1.679095E+02
164.00	1.924523E+00	-1.284941E+00	-1.924523E+00	-1.484870E+00	-2.459512E-01	2.265330E+00	-1.705950E+02
168.00	1.845689E+00	-1.273328E+00	-1.845689E+00	-1.476075E+00	-1.859919E-01	2.213390E+00	-1.728183E+02
172.00	1.788619E+00	-1.264745E+00	-1.788619E+00	-1.469850E+00	-1.420901E-01	2.180649E+00	-1.744784E+02
176.00	1.754077E+00	-1.259478E+00	-1.754077E+00	-1.466139E+00	-1.153132E-01	2.162861E+00	-1.755029E+02
180.00	1.742513E+00	-1.257703E+00	-1.742513E+00	-1.464906E+00	-1.063144E-01	2.157253E+00	-1.758491E+02

********** ACCUMULATED SUMS FOR N = 3 **********

CLASS 2

ANGLE	SCATTERING	TOTAL(REAL)	TOTAL(IMAG)	RTRAD(REAL)	RTRAD(IMAG)	RCS	PHASE ANGLE
.00	1.742513E+00	1.257703E+00	1.742513E+00	-1.150645E+00	-8.465618E-01	2.040651E+00	-1.436570E+02
4.00	1.744547E+00	1.259799E+00	1.744547E+00	-1.153900E+00	-8.468857E-01	2.048700E+00	-1.437238E+02
8.00	1.750623E+00	1.266054E+00	1.750623E+00	-1.163619E+00	-8.478370E-01	2.072638E+00	-1.439222E+02
12.00	1.760666E+00	1.276368E+00	1.760666E+00	-1.179673E+00	-8.493558E-01	2.113033E+00	-1.442464E+02
16.00	1.774549E+00	1.290571E+00	1.774549E+00	-1.201838E+00	-8.513454E-01	2.169203E+00	-1.446874E+02
20.00	1.792090E+00	1.308430E+00	1.792090E+00	-1.229804E+00	-8.536772E-01	2.241183E+00	-1.452333E+02
24.00	1.813049E+00	1.329644E+00	1.813049E+00	-1.263171E+00	-8.561967E-01	2.328675E+00	-1.458700E+02
28.00	1.837127E+00	1.353852E+00	1.837127E+00	-1.301448E+00	-8.587292E-01	2.431183E+00	-1.465821E+02
32.00	1.863959E+00	1.380630E+00	1.863959E+00	-1.344054E+00	-8.610864E-01	2.547952E+00	-1.473538E+02
36.00	1.893119E+00	1.409499E+00	1.893119E+00	-1.390323E+00	-8.630714E-01	2.677891E+00	-1.481692E+02
40.00	1.924117E+00	1.439931E+00	1.924117E+00	-1.439506E+00	-8.644830E-01	2.819508E+00	-1.490134E+02
44.00	1.956403E+00	1.471356E+00	1.956403E+00	-1.490780E+00	-8.651184E-01	2.970855E+00	-1.498729E+02
48.00	1.989374E+00	1.503170E+00	1.989374E+00	-1.543261E+00	-8.64774[illegible]-01	3.12948[illegible]E+00	-1.507357E+02
52.00	2.022384E+00	1.534750E+00	2.022384E+00	-1.596014E+00	-8.632462E-01	3.292453E+00	-1.515921E+02
56.00	2.054756E+00	1.565462E+00	2.054756E+00	-1.648076E+00	-8.603300E-01	3.456322E+00	-1.524345E+02
60.00	2.085795E+00	1.594679E+00	2.085795E+00	-1.698476E+00	-8.558192E-01	3.617246E+00	-1.532576E+02
64.00	2.114813E+00	1.621793E+00	2.114813E+00	-1.746258E+00	-8.495961E-01	3.771079E+00	-1.540586E+02
68.00	2.141141E+00	1.646230E+00	2.141141E+00	-1.790512E+00	-8.411837E-01	3.913522E+00	-1.548358E+02
72.00	2.164156E+00	1.667469E+00	2.164156E+00	-1.830393E+00	-8.306491E-01	4.040317E+00	-1.555910E+02
76.00	2.183301E+00	1.685048E+00	2.183301E+00	-1.865157E+00	-8.177094E-01	4.147461E+00	-1.563267E+02
80.00	2.198099E+00	1.698584E+00	2.198099E+00	-1.894179E+00	-8.021906E-01	4.231423E+00	-1.570472E+02
84.00	2.208179E+00	1.707778E+00	2.208179E+00	-1.916974E+00	-7.839473E-01	4.289361E+00	-1.577579E+02
88.00	2.213285E+00	1.712426E+00	2.213285E+00	-1.933214E+00	-7.628746E-01	4.319292E+00	-1.584651E+02
92.00	2.213285E+00	1.712426E+00	2.213285E+00	-1.942738E+00	-7.389203E-01	4.320232E+00	-1.591757E+02
96.00	2.208179E+00	1.707778E+00	2.208179E+00	-1.945553E+00	-7.120960E-01	4.292257E+00	-1.598968E+02
100.00	2.198099E+00	1.698584E+00	2.198099E+00	-1.941833E+00	-6.824868E-01	4.235504E+00	-1.606352E+02
104.00	2.183301E+00	1.685048E+00	2.183301E+00	-1.931907E+00	-6.502574E-01	4.155100E+00	-1.613974E+02
108.00	2.164156E+00	1.667469E+00	2.164156E+00	-1.916244E+00	-6.156552E-01	4.051022E+00	-1.621887E+02
112.00	2.141141E+00	1.646230E+00	2.141141E+00	-1.895433E+00	-5.790087E-01	3.927918E+00	-1.630134E+02
116.00	2.114813E+00	1.621793E+00	2.114813E+00	-1.870161E+00	-5.407211E-01	3.789883E+00	-1.636738E+02
120.00	2.085795E+00	1.594679E+00	2.085795E+00	-1.841187E+00	-5.012600E-01	3.641230E+00	-1.647704E+02
124.00	2.054756E+00	1.565462E+00	2.054756E+00	-1.809315E+00	-4.611436E-01	3.486275E+00	-1.657013E+02
128.00	2.022384E+00	1.534750E+00	2.022384E+00	-1.775375E+00	-4.209236E-01	3.329134E+00	-1.666620E+02
132.00	1.989374E+00	1.503170E+00	1.989374E+00	-1.740196E+00	-3.811675E-01	3.173571E+00	-1.676452E+02
136.00	1.956403E+00	1.471356E+00	1.956403E+00	-1.704589E+00	-3.424400E-01	3.022888E+00	-1.686409E+02
140.00	1.924117E+00	1.439931E+00	1.924117E+00	-1.669330E+00	-3.052860E-01	2.879862E+00	-1.696363E+02
144.00	1.893119E+00	1.409499E+00	1.893119E+00	-1.635149E+00	-2.702158E-01	2.746729E+00	-1.706164E+02
148.00	1.863959E+00	1.380630E+00	1.863959E+00	-1.602720E+00	-2.376934E-01	2.625208E+00	-1.715642E+02
152.00	1.837127E+00	1.353852E+00	1.837127E+00	-1.572652E+00	-2.081282E-01	2.516551E+00	-1.724612E+02
156.00	1.813049E+00	1.329644E+00	1.813049E+00	-1.545489E+00	-1.818709E-01	2.421612E+00	-1.732884E+02
160.00	1.792090E+00	1.308430E+00	1.792090E+00	-1.521702E+00	-1.592123E-01	2.340926E+00	-1.740270E+02
164.00	1.774549E+00	1.290571E+00	1.774549E+00	-1.501693E+00	-1.403859E-01	2.274791E+00	-1.746592E+02
168.00	1.760666E+00	1.276368E+00	1.760666E+00	-1.485791E+00	-1.255717E-01	2.223342E+00	-1.751691E+02
172.00	1.750623E+00	1.266054E+00	1.750623E+00	-1.474249E+00	-1.149017E-01	2.186613E+00	-1.755434E+02
176.00	1.744547E+00	1.259799E+00	1.744547E+00	-1.467251E+00	-1.084654E-01	2.164591E+00	-1.757721E+02
180.00	1.742513E+00	1.257703E+00	1.742513E+00	-1.464906E+00	-1.063144E-01	2.157253E+00	-1.758491E+02

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13. ABSTRACT

The purpose of this work is to describe a theoretical formulation, including a documented computer program, for the evaluation of electromagnetic scattering by perfectly conducting bodies having an axis of rotational symmetry. The main body of the work gives the theory, which has been modified considerably from that given earlier. Appendix I gives the analysis and logic which forms the basis for the various subroutines of the computer program. Appendix II gives the complete FORTRAN listings of the computer program. Finally, Appendix III gives the computer printout for a numerical example, scattering by a conducting sphere-cone-sphere obstacle, as obtained on the IBM 7030 digital computer.

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